International Journal of Innovative Mathematics, Statistics & Energy Policies
13(1):59-65, Jan.-Mar., 2025

 www.seahipublications.org ISSN: 2467-852X

doi:10.5281/zenodo.14808759

Linear Programming an Optimization Technique for optimal production planning: A case of Abdu Gusau Polytechnic Pure water production, Talata Mafara.

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ABSTRACT
Linear Programming Problem (LPP) as an applied mathematical model has been applied in business problem and related discipline for optimization (maximizing profit/contribution or minimizing cost/time of production). This research work applied linear programming model as an optimal model on the production planning for the maximization of profit/contribution in Abdu Gusau Polytechnic, Talata Mafara, Zamfara State Nigeria. Linear programming approach was adopted in order to maximize profit. The data for the study was secondary data collected from the company by interview method. The variables of the study were the machine and the labour available hours. These data were formulated into the linear programming model and analyzed using LINGO software. The optimal tableaus of the LINGO iterations were obtained. The results revealed that the company should produce 195 quantities of pure water (X_1) from machine 1 and 150 quantities of pure water (X_2) from machine 2 on a daily basis in order to yield daily profit of $Z = N18{,}000.00k$ in order to gain much profit.

Keywords: Optimization, Linear Programming Model, Water, Profit, Production, LINGO.

INTRODUCTION

Optimization has become a common phenomenon in almost all organizations and establishments. In developed, countries managerial decisions are mostly based on the use of optimization techniques. In a profit making organizations, the Board of Directors has many things to tackle which may include: The problem of other competitors in the same business, availability of funds for new capital projects, reduction of operational cost, high level of output and ultimately maximization of profit as explained by Nonso (2005).

Application of linear programming to a production company is very important as its guide the management of the company to make optimal decision and maximize profit (Danfulani, et al, 2022)

In an attempt to address these problems there are two techniques of operation that may be applied. They include: The quantitative technique and the qualitative technique. Quantitative technique which is preferred involves modelling of a ‘real form’ problem into a mathematical form which can be solved to arrive at a Solution that would aid the decision makers. Linear programming (LP) technique is such a Quantitative technique: It is a widely used Mathematical modelling technique concerned with the efficient allocation of limited resources to known activities with the objective of meeting the desired goal (Taha, 1977).

In 1947, during World War II, George B. Dantzing while working with the US Air force, developed LP model primarily for solving military logistics problems. But now, it is extensively being used in all functional areas of management, airlines, agriculture, military operations, oil refining, education, energy planning, health care system etc.

Linear programming (LP, also called linear optimization) is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. Linear programming is a special case of mathematical programming (also known as mathematical optimization). More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints. Its feasible region is a convex polytope, which is a set defined as the intersection of finitely many half spaces, each of which is defined by a linear inequality. Its objective function is a real-valued affine (linear) function defined on this polyhedron. A linear programming algorithm finds a point in the polytope where this function has the smallest (or largest) value if such a point exists.

Linear programs are problems that can be expressed in canonical form as:

$Mazimize\ Z = C^{T}X$

$Subject$ to $AX \leq b$

$and\ X \geq 0$

Where **x** represents the vector of variables (to be determined), **c** and **b** are vectors of (known) coefficients, *A* is a (known) matrix of coefficients, and $\mathbf{c}^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($\mathbf{c}^T\mathbf{x}$ in this case).

The inequalities $A\mathbf{x} \leq \mathbf{b}$ and $\mathbf{x} \geq \mathbf{0}$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions. If every entry in the first is less-than or equal-to the corresponding entry in the second, then it can be said that the first vector is less-than or equal-to the second vector. Generally, s given as follows:

$Maximize\ Z = C_1x_1 + C_2x_2 + \ldots C_nx_n$

Subject to:

$a_{11}x_1 + a_{22}x_2 + \cdots + a_{1n}x_n \leq b_1$

$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_nn \leq b_2$

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$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \leq b_n$

$x_1 \geq 0, \quad x_2 \geq 0 \ldots x_n \geq 0$

The above is called the standard form. The terms:

$C_1, C_2, C_3 \ldots C_n, b_1, b_2, b_3 \ldots b_n$ and $a_{11}, a_{12}, a_{13} - - - - - - a_{1n}$, are given numbers and $x_1, x_2, x_3 \ldots x_n$ are variables whose values are to be determined (decision Variables).

In this case, the function being maximized i.e $C_1x_1 + C_2x_2 + \ldots C_n x_n$, is called the OBJECTIVE FUNCTION and the restrictions normally referred to as the CONSTRAINTS..

In allocation of limited resourses to the best advantage, Linear programming is not a new phenomenon in allocation of scarce resources and achieving optimum decision among competing activities. Linear programming (LP) according to Miller (2007) is a generalization of linear algebra use in modeling so many real life problems ranging from scheduling of Airline routes to shipping oil from refineries to cities for the purpose of finding inexpensive diet capable of meeting daily requirement. Miller argues that the reason for the great versatility of linear programming is due to ease at which constraints can be incorporated in to the linear programming model. Linear programming method has proven, given the raw data the only way the company production problem can be addressed (A.E. Anieting, 2013).

Using a mathematical model, we evaluate all possible combinations to find the one that satisfies product specifications at the lowest price. Mathematical modeling is quicker and less expensive than using the trial-and-error approach and constrained optimization models are mathematical models that find the best solution with respect to some evaluation criterion from a set of alternative solutions. (.Hillier,1986). Linear programming is an important branch of applied mathematics that solves a wide variety of optimization problems. It is widely used in production planning and scheduling problems and the biggest advantage of linear programming as an optimization method is that it always achieves the optimal solution if one exists (Strang, 1988). Frizzone, et al (1997) used linear programming model to optimize the water resource used in irrigation, and obtained an optimal way of water resource usage. Though, considering the numerous constraints, he had to develop a separate mathematical model to achieve his goal. He noted that the mathematical programming quantifies an optimal way of combing scarce resources to satisfy the proposed goals; that is to analyze the cases where the available resources must be combined in a way to maximize the profit or minimize cost.

Nonso (2005) in his work on Application of Linear programming for Managerial Decision found out how an organization can have effective control over materials for input during production. He observed that units produced must be assumed as what is sold in order to achieve the company's goal.

Fagoyinbo, and Ajibode, (2010) worked on the Application of Linear programming Techniques in the Effective use of resources for staff training. The method employed gave an integer optimum solution to all the models formulated. The Data used did not yield a feasible solution but when the model reformed gave an optimum solution. Linear programming method has proven, given the raw data the only way the company production problem can be addressed (Olubiyi et al, 2021). Ezema and Amakom (2012) worked on optimizing profit with the linear programming model: A focus on Golden plastic Industry limited. Enugu. 2012. The result they had showed that only 2 sizes of the total 8 'PVC' pipes should be produced. According to the study's findings, dependability modeling may be used to assess the strength, efficiency, and performance enhancement of a reverse osmosis (RO) system. The current work will be beneficial to water manufacturing and industrial uses that are dangerous to humans, among other things (Maihulla, 2022).

Statement of the Problem/justification

AGP Pure water center which was established in the year 2011 with aim of making profit as well as training of students as entrepreneur center. All production industries /firms' aim at maximizing profit after sales and Linear programming method has proven to be one of the way the company production problem can be addressed (Olubiyi et al, 2021). The AGP pure water production center has two production machines that produces the sachet water which is required to determine the optimal number of sachets water for a given time period that will give the company the optimum profit. Amakom (2012) worked on optimizing profit with the linear programming model. Failures for the AGP center to efficiently allocate the limited resources to the best advantage and optimize the objective function under the available constraints can lead to company run in to loos and if care is not taken can results to collapse, hence, the needs for the research.

Objectives

The main objective of this study is to formulate the Linear programming problem (LPP) model for the center's available resources and constraints. Hence, it set to:

1. Formulate the linear programming problem for the data obtained
2. Determined the optimum units of sachet water from the two production machine that maximize the stated objective function
3. To Estimate the optimum profit for the production center.

METHODOLOGY

The study is a quantitative one in which secondary data were obtained through interview method in which questions regarding to the number of machines with their respective production capacity per day and their time capacity in hours, number of staff with their daily available hours which will form the linear programming problem constraints equations, and the contributions of the products from the two machines (sachet pure water produce from machine 1 from machine 2) to form the LPP objective function in formulation of LPP. LINGO software was used in solving the formulated LPP to determine the optimum solutions

Model Assumptions

The raw materials required for productions of pure water are assumed limited and the qualities of variables used in water production are also assumed o be standard.

The machine and labour hours for the production of sachet water are assumed limited it is also assumed that an effective allocation of the variables used (sachet bag of pure water (50cl) will aid optimal production and at the same time maximizing the profit of the AGP table water.

In general the decision variables must be continuous; they can take on any value within some restricted range, the objective function must be a linear function and the left-hand sides of the constraints must be linear functions.

DATA PRESENTATION

The Abdu Gusau Polytechnic, Talata Mafara Pure water production center manufactures /produces pure water from two different machines leveled as products $X_1$from machine 1 and X_2 from machine 2.The product from machine 1 are reserved for customers coming to the production center sold at lower price while product from machine 2 are taken outside and sold at a price higher than that of machine 1 To produce one batch of 100 bags as a unit of X_1, 1 machine hour and 1.5 labour hours are required. To produce one batch of 200 bags productX_2, 1.5 machine hours and 2.5 labour hours are required. In a month, 420 machine hours and 540 labour hours are available. Profit estimated per unit for X_1 is ₦50 for a batch and estimated for X_2 is ₦55 for a batch. Formulate as LPP.

Products	Resource/unit		Estimated Profit
	Machine	Lobour	
X_1	1	1	₦50
X_2	2	1.5	₦55
Availability	420hrs/Month	540hrs/Month	

There will be two constraints. One for machine hour's availability and for labour hours availability.

Model formulation:

Decision variables

X_1 = Number of units of sachet pure water produced from machine 1.

X_2 = Number of units of sachet pure water produced from machine 2

Model formulation:

The objective function:

$$MaxZ = 50X_2 + 55X_2 \quad (1)$$

Subjective Constraints

For machine hours

$$2X_1 + X_2 \leq 420 \quad (2)$$

For labour hours

$1X_1 + 1.5X_2 \leq 540$ (3)

Non negativity

$X_1, X_2 \geq 0$ (4)

DATA ANALYSIS AND RESULTS

Model LINGO code: Below is the code that produces the results for the formulated linear programming problem of the AGP pure water center.

```
!Objective Function;
Max = 50*X1 + 55*X2;
!Subject to constraints;
X1 >= 0;
X2 >= 0;
!Equations;
2*X1 + 1*X2 <= 540;
1*X1 + 1.5*X2 <= 420;
```

Results: LINGO/WIN32 20.0.10 (20 Dec 2022), LINDO API 14.0.5099.197

Global optimal solution found.

Objective value:	18000.00
Infeasibilities:	0.000000
Total solver iterations:	2
Elapsed runtime seconds:	0.10
Model Class:	LP
Total variables:	2
Nonlinear variables:	0
Integer variables:	0
Total constraints:	5
Nonlinear constraints:	0
Total nonzeros:	8
Nonlinear nonzeros:	0

Variable	Value	Reduced Cost
X_1	195.0000	0.000000
X_2	150.0000	0.000000

Row	Slack or Surplus	Dual Price
1	18000.00	1.000000
2	195.0000	0.000000
3	150.0000	0.000000
4	0.000000	10.00000
5	0.000000	30.00000

Results summary:

Variables	Optimum value
X_1 (Number of bags/batch from machine 1)	195
X_2 (Number of bags/batch from machine 2)	150
Z (Maximum profit)	₦18000

DISCUSSION

The results of the analysis shows that the decision variables X_1 and X_2, that is the number of bags of pure water to be produce under the constraints of the available labour and machine hours for the production of pure water from the two different machines are 195 bags from machine 1 and 150 bags from machine 2 for each batch of the daily production which will enable the tier to have an optimum (maximum) profit of ₦18000 daily, hence, the best production schedule.

CONCLUSION

Application of Linear Programing to a production industry is very important as its guide the management of the industry to make optimal decision and maximize profit. In this research, we applied linear programming problem to Abdu Gusau Polytech Talata Mafara and found out that if the management of the firm can use the model in her production process and in consideration of the available time constrain of machine and labour; the center can make daily profit of N 18000.00k by producing 195 bags from machine 1 and 150 bags from machine 2. Also the analysis carried out in this research and the result indicates, AGP pure water should produce the pure water (sachet bag of 50cl pack from the two machines) in order to satisfy her customers outside and base community but, more of sachet bag of pure water should be produce from machine 1in order to attain maximum profit, because it contribute mostly to the profit earned by the center. Also linear programming method has proven, given the raw data the only way the company production problem can be addressed

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