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Magneto-hydrodynamic Heat Transfer of Power-Law Temperature-Dependent Surface Heat Flux in the Presence of Thermal Radiation

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ABSTRACT
The study observes the steady and transient Magneto-hydrodynamic (MHD) heat transfer of power law temperature-dependent surface heat flux in the present of thermal radiation and convective boundary condition flow within vertical walls. The models consider the silent features of nonlinear convective transport. The heat transfer mechanism involves the influences of a magnetic field, Prandtl number, power law temperature and non-linear radiation parameter as well as temperature difference. The convective condition is also retained at one boundary while the other boundary is non-convective. The non-linear steady and time dependent energy and momentum equations under relevant initial and boundary conditions are solved both analytically and numerically using perturbation and implicit finite difference respectively. The numerical upshots were validated with that of steady state version and an excellent agreement was found between transient and steady state solution at large value of time t. To ensure the strength and stability of the parameters, a sensitivity analysis was performed to understand how variation in different parameters affect the resulting temperature distribution or heat transfer. The heat transfer rate within the channel is significantly been influence by the power law temperature, convective (Biot number) parameter. And thermal radiation parameter. In addition the present outcomes are in agreement with the existing literature by direct appraisal.
Keywords: Magneto hydrodynamics, Power law temperature, Convective boundary condition, thermal radiation, vertical wall, steady/transient

INTRODUCTION

Magneto-hydrodynamic (MHD) heat transfer is a pivotal area of study within fluid dynamics, significantly impacting engineering and industrial applications where electromagnetic fields influence fluid flow and thermal behaviour. This branch of science merges principles from electromagnetism and fluid mechanics to explore the movement and heat transfer characteristics of electrically conductive fluids in the presence of magnetic fields. The applications of MHD are vast, ranging from nuclear reactors and metallurgical processes to cooling systems for electronic devices and even astrophysical phenomena (Nor *at el.* 2020). MHD flow plays important roles in the field of medicine, for example in cancer tumor treatment causing hypothermia, reducing bleeding in severe injuries and magnetic resonance imaging (Rashad 2017)

In particular, the investigation of MHD heat transfer in non-Newtonian fluids, such as those following a power-law model, adds another layer of complexity and relevance. Non-Newtonian fluids exhibit varying viscosity based on the rate of shear strain, which profoundly affects their flow and thermal profiles under

the influence of a magnetic field. Power-law fluids, characterized by their shear-thinning or shear-thickening behavior, are commonly encountered in various industrial processes, including polymer engineering, food processing, and biomedical applications. Among such non-Newtonian fluids, some fluids such as commercial carboxyl methyl cellulose in water, cement rock in water, napalm in kerosene, lime in water and Illinois yellow clay in water are power-law fluid (Satya *at el*. 2019)

An intriguing aspect of MHD heat transfer is the role of temperature-dependent surface heat flux, which aligns with real-world scenarios where surface conditions fluctuate with temperature changes. This dynamic condition necessitates a comprehensive understanding of how heat flux variations influence the overall heat transfer and fluid flow dynamics.

Furthermore, the presence of thermal radiation introduces additional complexity. Thermal radiation, which involves the transfer of heat through electromagnetic waves, becomes significant at high temperatures and can alter the thermal energy distribution within the fluid. The interplay between conductive, convective, and radiative heat transfer mechanisms in an MHD context provides a rich ground for exploration and optimization.

In recent decades heat transfer enhancement has been an interesting area due to its numerous applications in industries and engineering. Ramana *et al*. (2015) discussed the influence of Heat and Mass Transfer Effects on MHD Natural Convective Flow Past an Infinite Vertical Porous Plate with Thermal Radiation and Hall Current. Sandhya *et al*. (2020) explored Heat and mass transfer effects on MHD flow past an inclined porous plate in the presence of chemical reaction. Asia *et al*. (2020) investigated Heat and Mass Transfer in MHD Flow of Micro polar Fluid over a Curved Stretching Sheet. Relative importance of temperature-dependent properties in non-Newtonian natural convection around curved surfaces was studied by Panel *et al* (2021). Falana (2021) discussed Velocity Slip Effect on MHD Power-Law Fluid over a Moving Surface with Heat Generation, Viscous Dissipation and Thermal Radiation. The importance of temperature-dependent properties combined with the power-law behavior of the fluid was studied numerically to elucidate the extent and significance of non-Boussinesq effects. Shankar *et al*. (2022) discussed thermal radiation impact on magneto-hydrodynamic heat transfer micro polar fluid flow over a vertical moving porous plate: A finite difference approach. Isah *et al*. (2022) deliberated On transient MHD heat transfer within a radiative porous channel due to convective boundary conditions. An upshot of slip velocity and thermal radiation on magneto hydrodynamic transient fluid flow through vertical walls was discussed by Bashiru *at el*. (2022). Seyed *et al*. (2023) studied the effect of thermal radiation on convective heat transfer in MHD boundary layer Carreau fluid with chemical reaction. Iterative Solutions for the Nonlinear Heat Transfer Equation of a Convective-Radiative Annular Fin with Power Law Temperature-Dependent Thermal Properties was investigated by Varun *et al*. (2023). Nora and Sivasankaran (2024) studied the influence of Entropy Generation and Thermal Radiation on Magneto-Convective Flow of Heat-Generating Hybrid Nano-Liquid in a Non-Darcy Porous Medium with Non-Uniform Heat Flux.

This study explores the magneto-hydrodynamic heat transfer of power-law fluids with temperature-dependent surface heat flux, considering the influence of thermal radiation. By incorporating the effects of electromagnetic fields, non-Newtonian fluid dynamics, and radiative heat transfer, the research seeks to clarify the complex interactions and resulting thermal properties. The findings of this study are significant for improving thermal management systems, optimizing industrial processes, and advancing the theoretical understanding of MHD phenomena in complex fluid systems.

Mathematical formulation

Consider the flow of steady/transient laminar, incompressible, electrically conducting fluid (MHD) through vertical walls in the presence of magnetic field of magnitude B_0, the x'-axis is taken in the direction of main flow along the plate and y'-axis is normal to the plate. At time $t' \leq 0$, both the fluid and plates are assumed to be at rest at constant temperature T_0. At time

$t' > 0$ the temperature of the plate placed at $y' = 0$ is convective and the temperature of other plate at a distance H from it, is constant. The flow is assumed laminar and fully developed. Under this assumption,

the boundary-layer momentum and energy equations for the problem in the dimensional form are respectively given below:

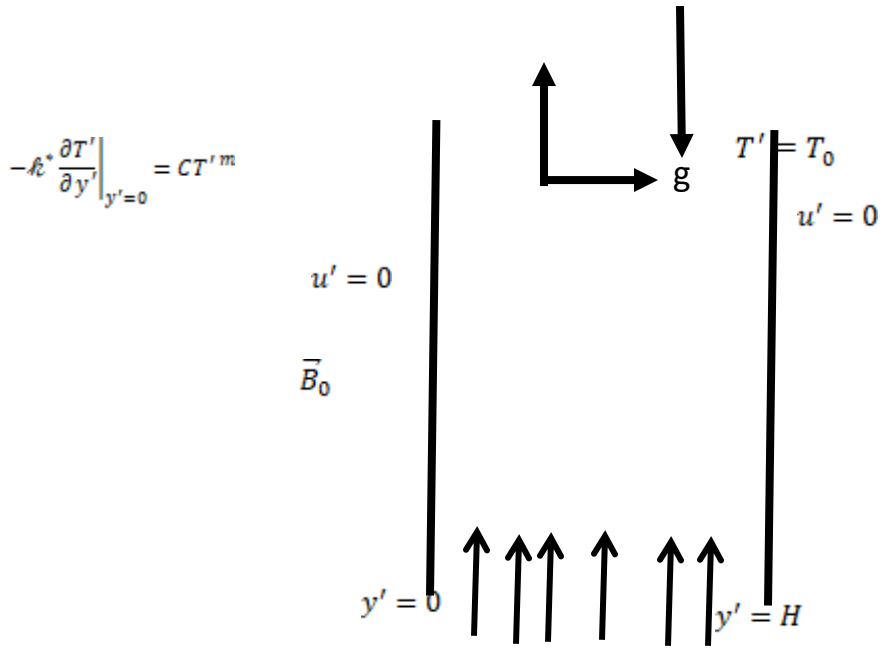

Figure 1: geometry of the problem for

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T_1 - T_0) - \sigma \frac{B_0{}^2 u'}{\rho} \tag{1}$$

$$\frac{\partial T'}{\partial t'} = \alpha \left[\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{K}\frac{\partial q_y}{\partial y'}\right] \tag{2}$$

The initial and boundary conditions to be satisfied are given as:

$$\left.\begin{aligned} t' \le 0: u' = 0; \quad T' = T_0 \;\; for \;\; 0 \le y' \le H \\ t' > 0: u' = 0, -k^* \frac{\partial T'}{\partial y'}\Big|_{y'=0} = CT'^m, at\ y' = 0 \\ u' = 0,\ T' = T_0, at\ y' = H \quad at\ y' = 1 \end{aligned}\right\} \tag{3}$$

The ration (q_y) of equation (2) signifies the radiation heat flux in the y'- direction while insignificant in the x'- direction relate to y'- direction. The ration term is predictably solved by using the Rosseland diffusion approximation for an optically thick fluid (Agha et al. 2014)

$$q_y = \frac{-4\sigma \partial T'^4}{3k^* \partial y'} \tag{4}$$

This estimate is binding at points optically far from the surface and good only for intensive absorption, which is for an optically thick boundary layer: It is expected that the temperature differences within the

flow involving the term T'^4 may be expressed as a linear function of temperature. Hence, expanding T'^4 result in

$$T'^4 = (\theta(T_1 - T_0) + T_0)^4 \tag{5}$$

The results of equations (1) and (2) subject to the boundary condition (3) in non-dimensional form can be established with the introduction of the succeeding dimensionless quantities

$$\left.\begin{aligned} t = \frac{t'\nu}{H^2},\quad y = \frac{y'}{H},\quad P_r = \frac{\nu}{\alpha}\ \theta = \frac{T'-T_0}{T_1-T_0}, B_{i1} = \frac{CH}{k^*}, G_r = \frac{[g\beta H^2(T_1-T_0)]}{\nu U}, \\ M^2 = \frac{\sigma\beta_0^2 H^2}{\nu}, u = \frac{u'}{U}, R = \frac{4\sigma(T_1-T_0)^3}{Kk^*}, C_T = \frac{T_0}{T_1-T_0} \end{aligned}\right\} \tag{6}$$

To obtain the required dimensionless momentum and energy equations, equations (5) with (6) are introduced into (1), and (2) subject to (3) such that:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta - M^2 u \tag{7}$$

$$P_r\frac{\partial \theta}{\partial t} = \left[1 + \frac{4R}{3}(C_T + \theta)^3\right]\frac{\partial^2\theta}{\partial y^2} + 4R[C_T + \theta]^2\left(\frac{\partial\theta}{\partial y}\right)^2 \tag{8}$$

With new initial and boundary conditions:

$$\left.\begin{aligned} t \le 0:\quad u = 0, \theta = 0\ , \text{for } 0 \le y \le 1 \\ t' > 0: u = 0, -\left.\frac{\partial\theta}{\partial y}\right|_{y=0} = B_{i1}\theta^m,\ at\ y = 0 \\ u = 0\ , \theta = 0,\ \ at\ y = 1 \end{aligned}\right\} \tag{9}$$

Solution procedure

Case 1: Analytical solution

To find the steady state version of MHD heat transfer flow within vertical walls due to convective boundary condition the terms associated with time derivative from equations (7) and (8), are set to be zero and reduces to:

$$\frac{d^2u}{dy^2} + G_r\theta - M^2u = 0 \tag{10}$$

$$\left[1 + \frac{4R}{3}(C_T + \theta)^3\right]\frac{d^2\theta}{dy^2} + 4R[C_T + \theta]^2\left(\frac{d\theta}{dy}\right)^2 = 0 \tag{11}$$

And the appropriate boundary conditions for equation (10) and (11) transform to:

$$\left.\begin{array}{l} at\ y = 0:\ u = 0, -\frac{d\theta}{dy} = B_{i1}\theta^m \\ at\ y = 1,\ u = 0, \theta = 0 \end{array}\right\} \quad (12)$$

Assuming the radiation parameter is so small $(R << 1)$ and taking power series expansion in the radiation parameter employs a regular perturbation method of the form representing the velocity (u), and temperature (θ) as:

$$u(y) = u_0(y) + Ru_1(y) + \cdots = \sum_{j=0}^{\infty} R^j u_j(y) \quad (13)$$

$$\theta(y) = \theta_0(y) + R\theta_1(y) + \cdots = \sum_{j=0}^{\infty} R^j \theta_j(y) \quad (14)$$

Substituting equations (10) and (11) respectively into (13) and (14) and equating the coefficient of like powers of R subject to boundary condition (12), the required solution for the governing momentum and energy equations are obtained as:

$$u(y) = a_3 e^{My} + a_4 e^{-My} a_5 y + a_6 + R(d_1 e^{My} + d_2 e^{-My} + k_1 y^4 + k_2 y^3 + k_3 y^2 + k_4 y + k_5) \quad (15)$$

$$\theta(y) = a_1 y + a_2 + R(c_1 y^4 + c_2 y^3 + c_3 y^2 + c_4 y + c_5) \quad (16)$$

At steady state the skin frictions at the boundaries are:

$$\tau_0 = \left.\frac{du}{dy}\right|_{y=0} = Ma_3 - Ma_4 + a_5 + Rd_1 M - Rd_2 M + Rk_4 \quad (17)$$

$$\tau_1 = \left.\frac{du}{dy}\right|_{y=1} = a_3 e^M - Ma_4 e^{-M} + a_5 + R(Md_1 e^M - Md_2 e^{-M} + 4k_1 + 3k_2 + 2k_3 + k_4) \quad (18)$$

The heat transfer on the flow walls are:

$$N_{u_0} = \left.\frac{d\theta}{dy}\right|_{y=0} = a_1 + Rc_4 \quad (19)$$

$$N_{u_1} = \left.\frac{d\theta}{dy}\right|_{y=1} = a_{11} + 4Rc_1 + 3Rc_2 + 2Rc_3 + c_4 \quad (20)$$

Case 2: Numerical solution

Since model of the present physical situation are coupled and highly non – linear due to presence of radiation effect. Therefore the time dependent transport equations of the present work are solved

numerically using finite difference method. Hence the transport equation 7 and 8 at grid point (i, j) are linearized to read as:
The momentum equation:

$$\frac{u_i^{j+1}-u_i^j}{\Delta t} = \left[\frac{u_{i-1}^{j+1}-2u_i^{j+1}+u_{i+1}^{j+1}}{(\Delta y)^2}\right] + G_r\theta_i^j - M^2u_i^j \tag{21}$$

The temperature equation:

$$P_r\frac{\theta_i^{j+1}-\theta_i^j}{\Delta t} = \left[1+\frac{4R}{3}\left(C_T+\theta_i^j\right)^3\right]\left[\frac{u_{i-1}^{j+1}-2u_i^{j+1}+u_{i+1}^{j+1}}{(\Delta y)^2}\right] + 4R\left[C_T+\theta_i^j\right]^2\left(\frac{\theta_{i-1}^j-\theta_{i+1}^j}{2\Delta y}\right)^2$$

(22)

With boundary condition

$$\left\{\begin{array}{ll} u_i^j = 0 & \\ -\left[\frac{-3\theta_{i-1}^{j+1}+4\theta_i^{j+1}-\theta_{i+1}^{j+1}}{2\Delta y}\right] = B_{i1}(\theta_i^j)^m & for\ all\ i=0 \\ u_M^j = 0, \theta_M^j = 0 & \end{array}\right\} \tag{23}$$

Further simplification (21) and (22) respectively give:

The velocity equation:

$$B_l u_{i-1}^{j+1} + B_c u_i^{j+1} + B_r u_{i+1}^{j+1} = (1-\Delta tM^2)u_i^j + \Delta tGr\theta_i^j \tag{24}$$

Temperature equation:

$$A_l\theta_{i-1}^{j+1} + A_c\theta_i^{j+1} + A_r\theta_{i+1}^{j+1} = P_r\theta_i^j + r_2\left[\theta_{i-1}^j - \theta_{i+1}^j\right]^2$$

(25)

Due to convective behaviour of the boundary of energy equation at $y = 0$, from equation (23) we have

$$\theta_{i-1}^{j+1} = \frac{4\theta_i^{j+1}-\theta_{i+1}^{j+1}}{3} + \frac{2}{3}\Delta yB_{i1}(\theta_i^j)^m \tag{26}$$

$$\theta_{i-1}^j = \frac{4\theta_i^j-\theta_{i+1}^j}{3} + \frac{2}{3}\Delta yB_{i1}(\theta_i^j)^m$$

(27)

Putting $i = 1$ into (26) and (27) and substituting into (25) and simplify, then the temperature equation becomes

$$\left(\frac{4}{3}Al + Ac\right)\theta_1^{j+1} + \left(Ar - \frac{1}{3}Al\right)\theta_2^{j+1} = -\frac{2}{3}Al\Delta yB_{i1}(\theta_1^j)^m + Pr\theta_1^j + r_2[\frac{4}{3}\theta_1^j - \frac{4}{3}\theta_2^j + \frac{2}{3}\Delta yB_{i1}(\theta_1^j)^m]^2$$

(28)

Using the known values of θ at grid point t = 0 and reducing the solution to tri-diagonal matrix we obtained temperature field at time $t_{i+1} = t_i + \delta t$ using the known values of the previous time $= t_i, for\ all\ i = 1, 2, \dots, M$. Then the velocity field is evaluated using the already known value of temperature field obtained at $t_{i+1} = t_i + \delta t$. These processes are repeated till the required solution of θ and u are gained at convergence criteria.

$$|(u,\theta)_{exact} - (u,\theta)_{num}| < 10^{-4} \quad (29)$$

The iterative system does not restrict time step and the technique is always convergent and unconditionally

Sensitivity analysis

To ensure the strength and stability of the parameters, a sensitivity analysis was performed to understand how variation in different parameters affect the resulting temperature distribution or heat flux. This is crucial in identifying which parameters most significantly influence heat transfer performance to improve the accuracy and reliability of thermal model by focusing on the critical factors. Positive and negative sensitivity analysis refer to how changes in input variable affect the output of a system or model I opposite direction. Understanding these differences is important in evaluating how sensitive a system is to changes in its parameters.

The sensitive analysis concerning the skin friction and heat transfer for the present study has been conducted on the radiation parameter R, temperature difference C_T, Grashof number Gr, magnetic parameter M, Biot number B_{i1}, power law temperature T_0, power law index n. To determine the sensitivity analysis of each parameter, the following equations are implied:

$$S\ A = \frac{\tau_0(X+\Delta X) - \tau_0(X-\Delta X)}{(X+\Delta X) - (X-\Delta X)} \quad (30)$$

$$S_X = \frac{SA}{\sum SA} \times 100 \quad (31)$$

Where $\Delta X = 10\%\ of\ X$ (32)

X	$Value\ of\ X$	$X + \Delta X$	$X - \Delta X$	$\tau_0(X + \Delta X)$	$\tau_0(X - \Delta X)$	$S.A$	$S_X\%$
R	0.1	0.11	0.09	0.1592	0.1303	1.445	101.4
M	1	1.1	0.9	0.1415	0.1477	- 0.031	-2.17
Gr	0.5	0.55	0.45	0.1478	0.1476	0.002	0.14
B_{i1}	0.5	0.55	0. 45	0.1481	0.1472	0.009	0.63

Table 1: 10% sensitivity analysis regarding skin friction (τ_0) at heated wall

X	$Value\ of\ X$	$X+\Delta X$	$X-\Delta X$	$Nu_0(X+\Delta X)$	$Nu_0(X-\Delta X)$	$S.A$	$S_X\%$
R	0.1	0.11	0.09	0.0791	0.0775	0.08	76.19
C_T	0.01	0.011	0.009	0.0782	0.0783	- 0.05	- 47.62
T_0	0.5	0.55	0.45	0.1478	0.1476	0.002	1.904
B_{i1}	0.5	0.55	0. 45	0.1481	0.1472	0.009	8/57
n	2	2.2	1.8	0.00396	0.0300	- 0.065	-61.904
M_0	2	2.2	1.8	0.0417	0.0279	0.0345	32.86

Table 1: 10% sensitivity analysis regarding Nusselt number (Nu_0) at heated wall

Validation of numerical solution:

It is observed from figure 2 (a) and (b) that analytical results obtained using perturbation method are compared with that of numerical results as time (t) approaches the value of the working fluid parameter (Pr). The result obtained reveals an excellent agreement between analytical and numerical results. This confirmed the validity of the numerical solution.

RESULTS AND DISCUSSION

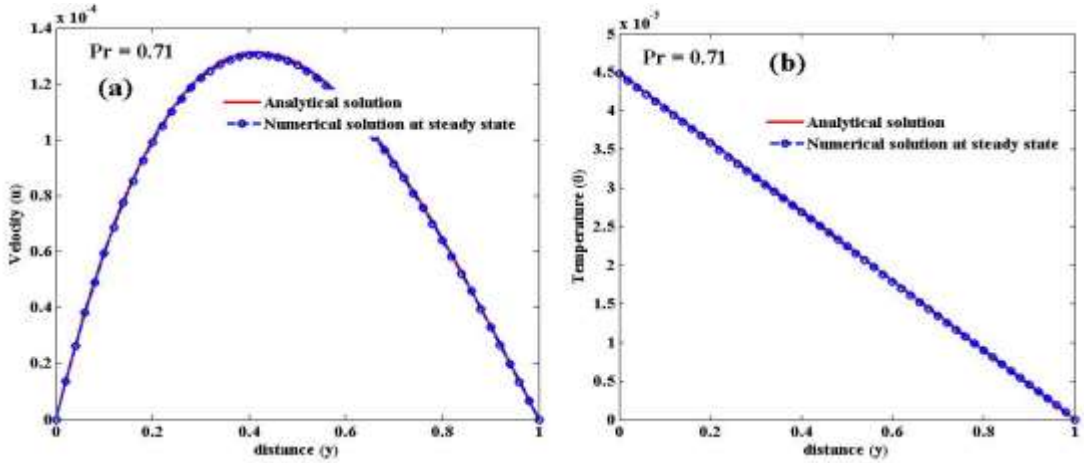

Figure 2: comparison of analytical and numerical solutions at steady state

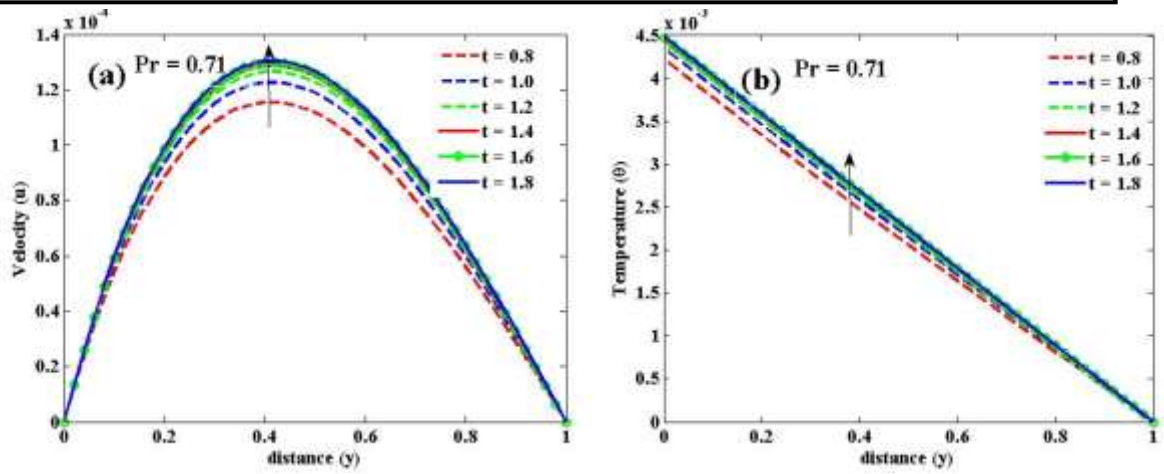

Figure 3: influence of time on steady state velocity and temperature

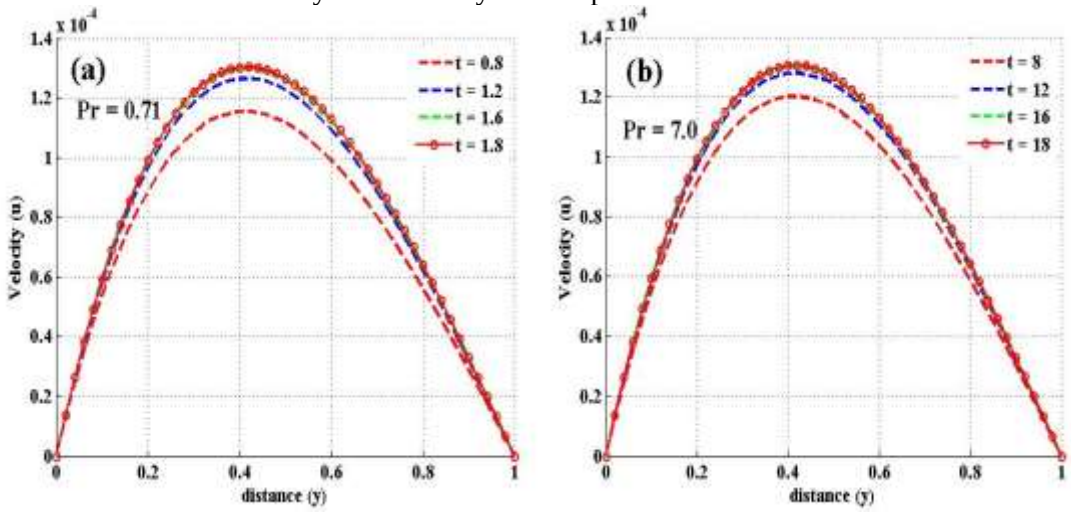

Figure 4: comparison of steady state solution for air and water at different time

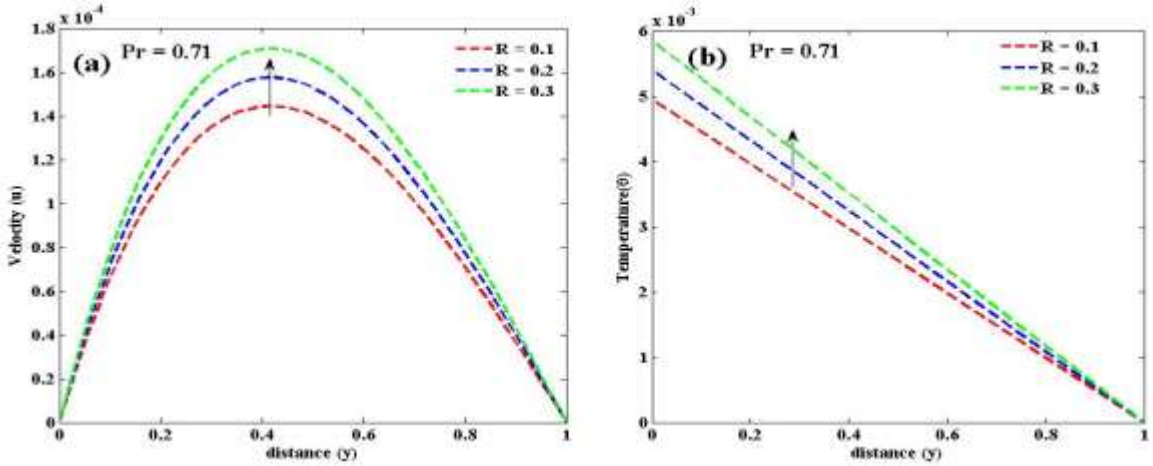

Figure 5: influence of R on velocity and temperature

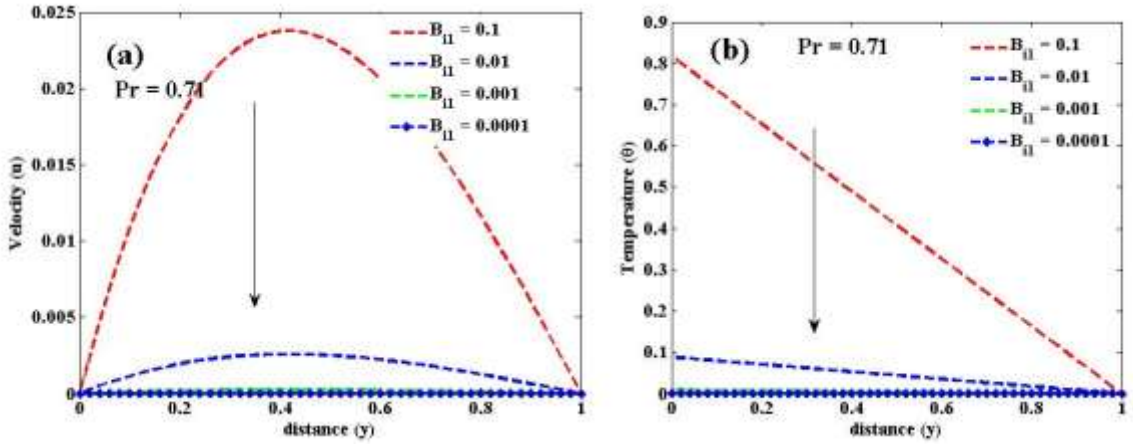

Figure 6: influence of B_{i1}on velocity and temperature

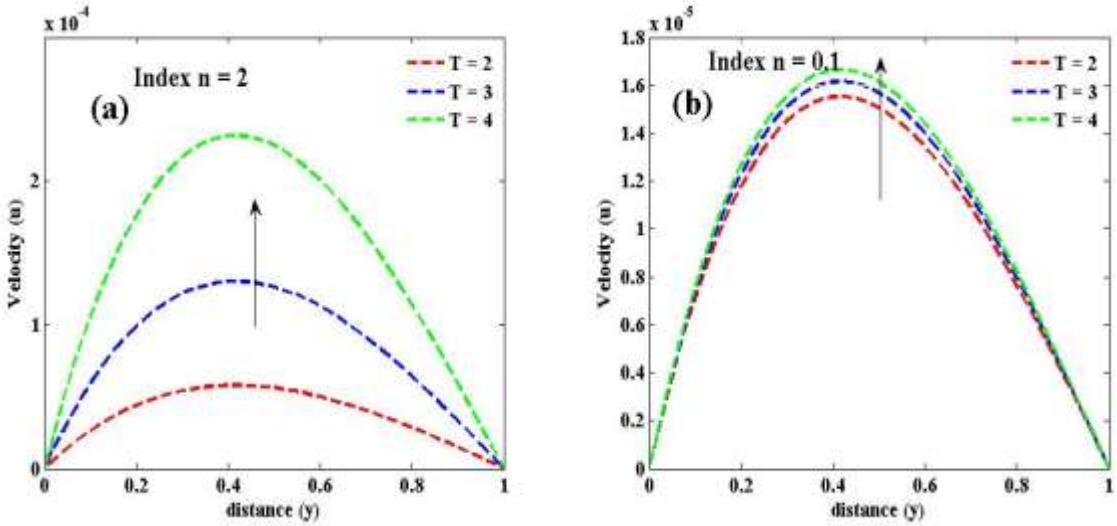

Figure 7: influence of power law temperature on velocity profile for different index

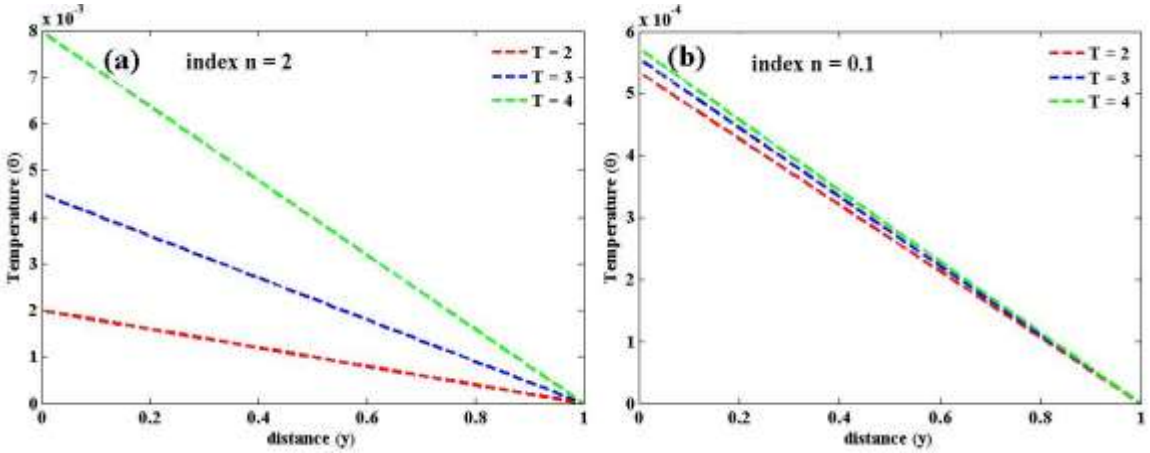

Figure 8: influence of power law temperature on temperature profile for different index

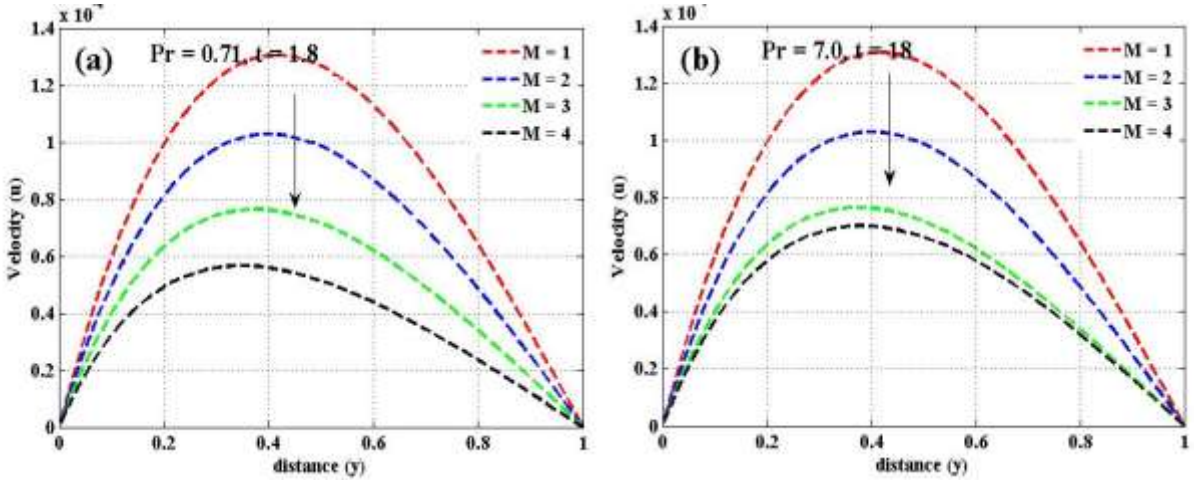

Figure 9: influence of M on velocity profile

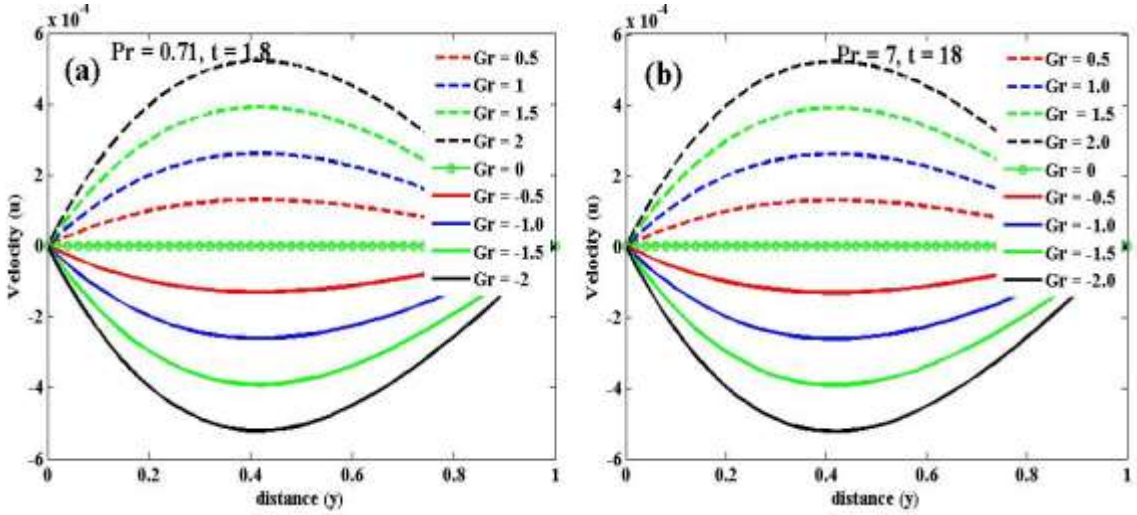

Figure 10: influence of Gr on velocity profile

DISCUSSION

Velocity and Temperature

Magneto-hydrodynamic (MHD) heat transfer of Power-Law temperature-dependent surface heat flux in the presence of thermal radiation have been evaluated within parallel plates. The boundary of the plate at y = 0 is convective while the other plate at y = 1 is non – convective. The thermal Roseland diffusion approximation was employed to represent the radiative heat flux, taking into account the temperature difference parameter. Validation of numerical scheme was performed using the analytical result obtained from perturbation method, showing excellent agreement between the numerical and analytical solutions with negligible thermal radiation parameter. In this research, parametric values were considered over a reasonable range.

$$0.001 \leq R \leq 0.1, 0.001 \leq C_T \leq 0.1, -0.2 \leq Gr \leq 2.0,\ 0.0001 \leq B_{i1} \leq 0.1, 0.8 \leq t \leq 18\ (for\ air\ and\ water), 0.71 \leq Pr \leq 7.0, 1 \leq M \leq 4, 1 \leq n \leq 3\,.$$

Figures 3 (a) and (b) depict the effect of time on velocity and temperature respectively to attain steady state. It is observed that in each case, steady state is attained when t is 1.8 units for air. Figures 4 (a) and (b) shows the effect of t on steady state for air and water respectively. It is observed that, the time it takes to attain steady state for water (Pr = 7.0) is twice that of air (Pr = 0.71). Figures 5 (a) and (b) illustrate the impact of radiation parameter, R on velocity and temperature profiles respectively. Velocity and temperature enhance with increase in R. figure 6 (a) and (b) depict the influence of Biot number on velocity and temperature respectively. It is observed from the figures that, as Biot number decreases the flow of velocity and temperature also decreases. . Figures 7 and 8 depict the impact of power law temperature and index on velocity and temperature respectively. Both velocity and temperature profiles upsurge with rise in power law temperature and index. It is witnessed that, the index plays an important role in the impact of power law temperature. When the index is 2, power law temperature has more significant effect then when it is 0.1 (see figures 7, 8 (a) and (b)). Figures 9 (a) and (b) illustrate the effect of magnetic parameter M on velocity profile. Due to drag force, velocity profile declines as M rises. Figure 10 shows the impact of Gr on velocity profile. Upsurge in Gr leads to rise in velocity profile.

Skin friction:

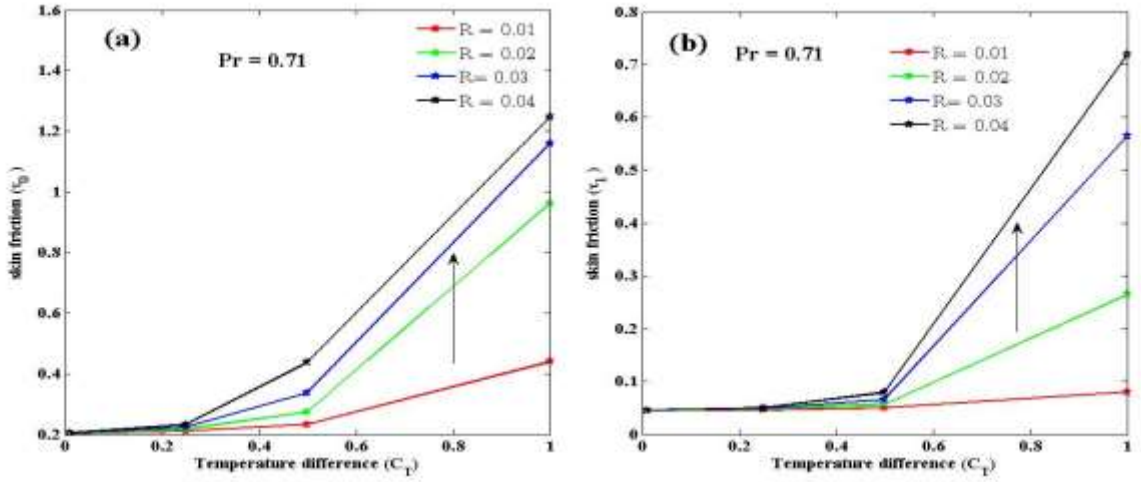

Figure 11: influence of C_T and R on skin friction at right and left walls for air $(Pr = 0.71)$

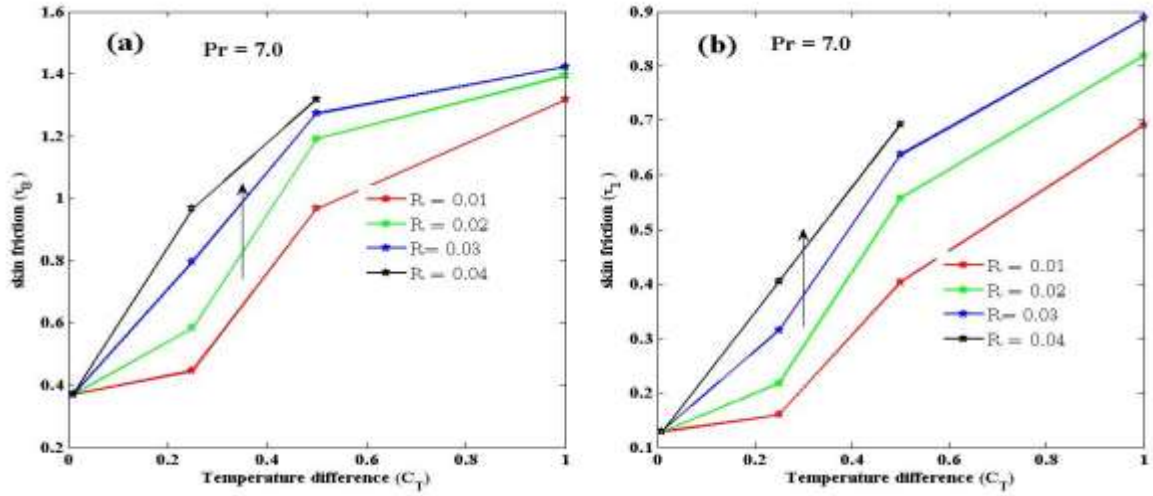

Figure 12: Influence of C_T and R on skin friction for water $(Pr = 7.0)$

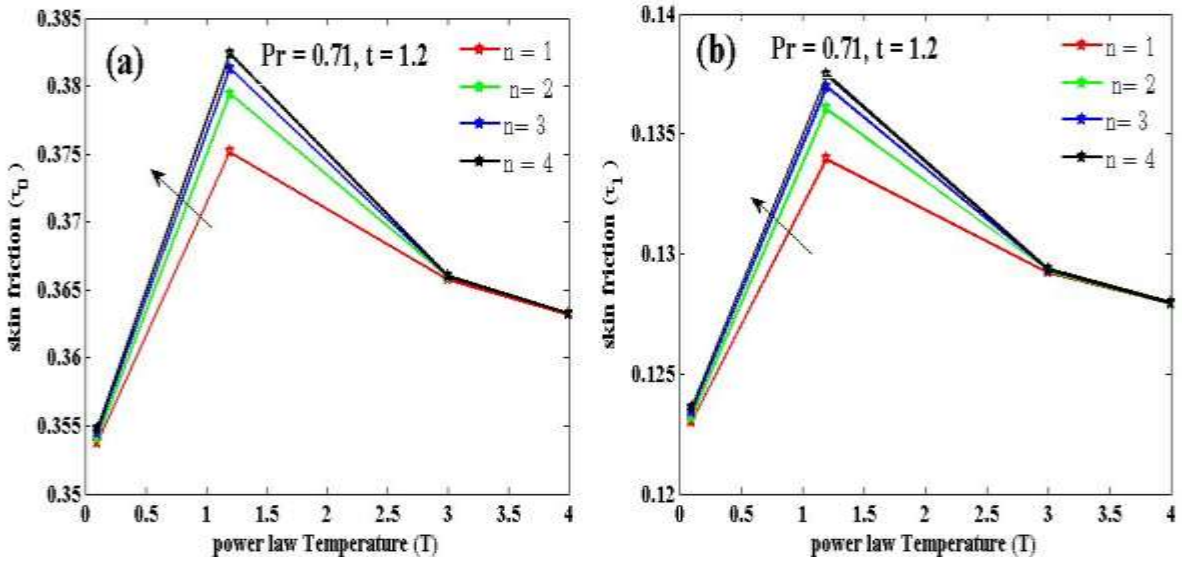

Figure 13: influence of power law temperature and index on skin friction for air

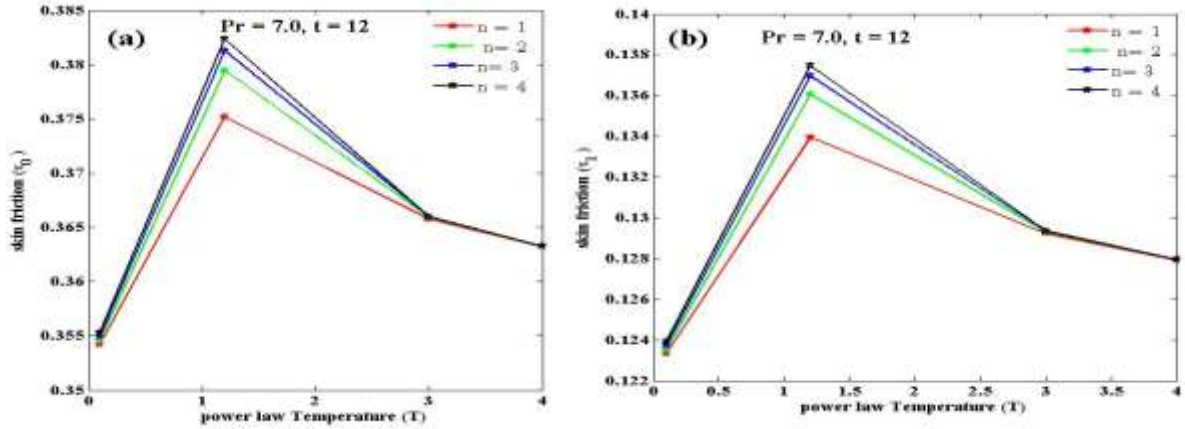

Figure 14: influence of power law temperature and index on skin friction for water for large t

Figures 11(a) and (b) show the effect of R and C_T for air (Pr = 0.71)on skin friction at convective and non – convective plates respectively. Skin friction rises as R and C_T increase. It also observed that, skin friction remain unchanged at certain value of C_T $(C_T < 0.2)$. Figures 12 (a) and (b) indicate the impact of C_T and R for water (Pr = 7.0). Increase in R and C_T leads to rise in skin friction. Figures 13 (a) and (b) indicate the impact of power law temperature and index on skin friction for air (Pr = 0,71). It is observed that at power law temperature $T < 1.5$, skin friction increases, but it decreases when $T > 1.5$. Figures 14 (a) and (b) depict the influence of power law temperature and index on skin friction for water (Pr = 7.0) at heated and cooled plates respectively. Skin friction upsurge as power law and index rise at both plates when $T < 1.5$. and decline when $T > 1.5$.

Nusselt number:

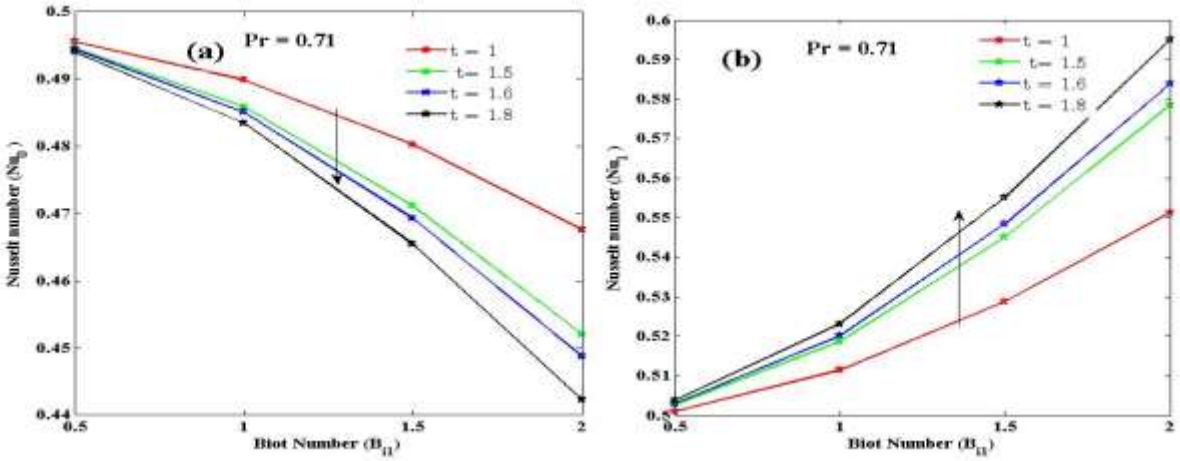

Figure 15: influence of time and B_{i1} on Nusselt number at heated and cooled plates

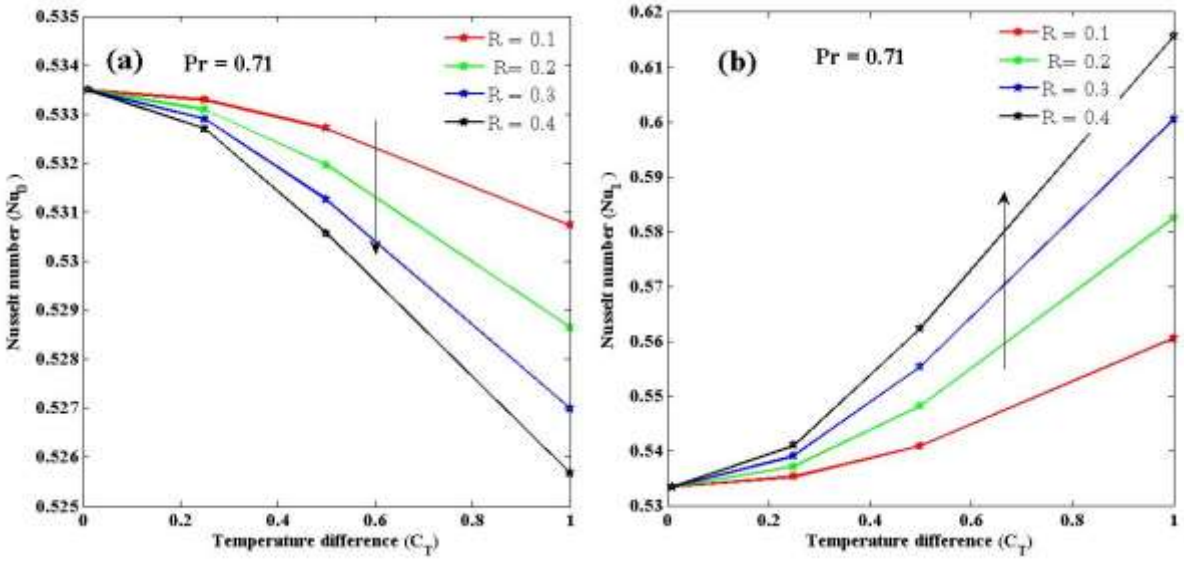

Figure 16: influence of R and C_T on Nusselt number at heated and cooled plates

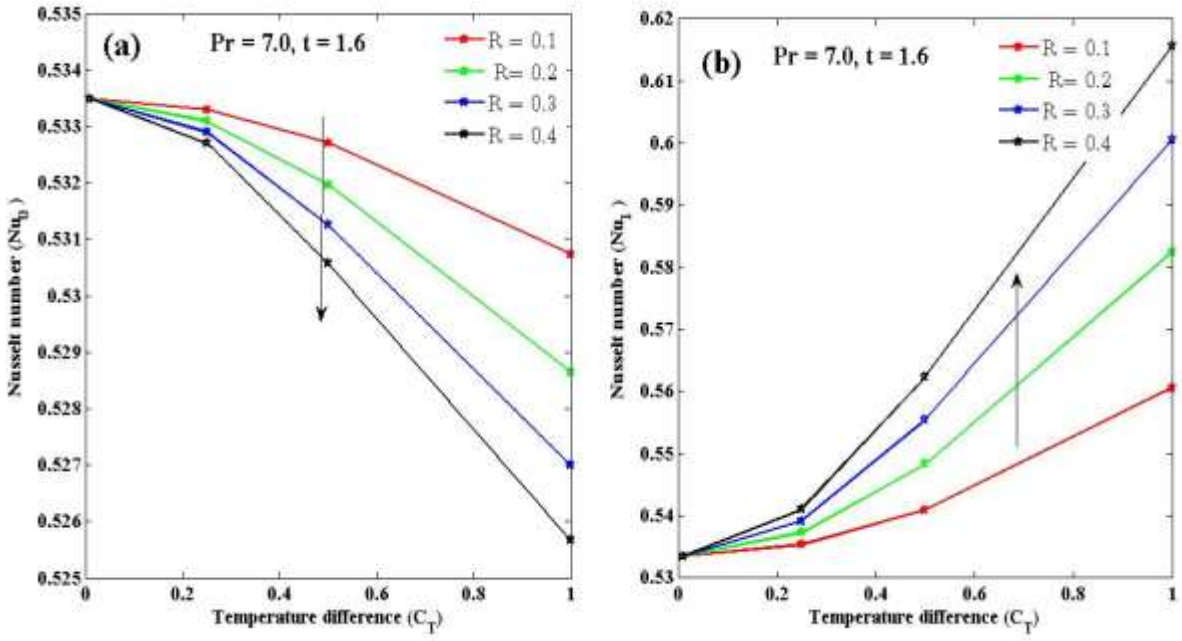

Figure 17: influence of R and C_T on Nusselt number at heated and cooled plates for water at small t

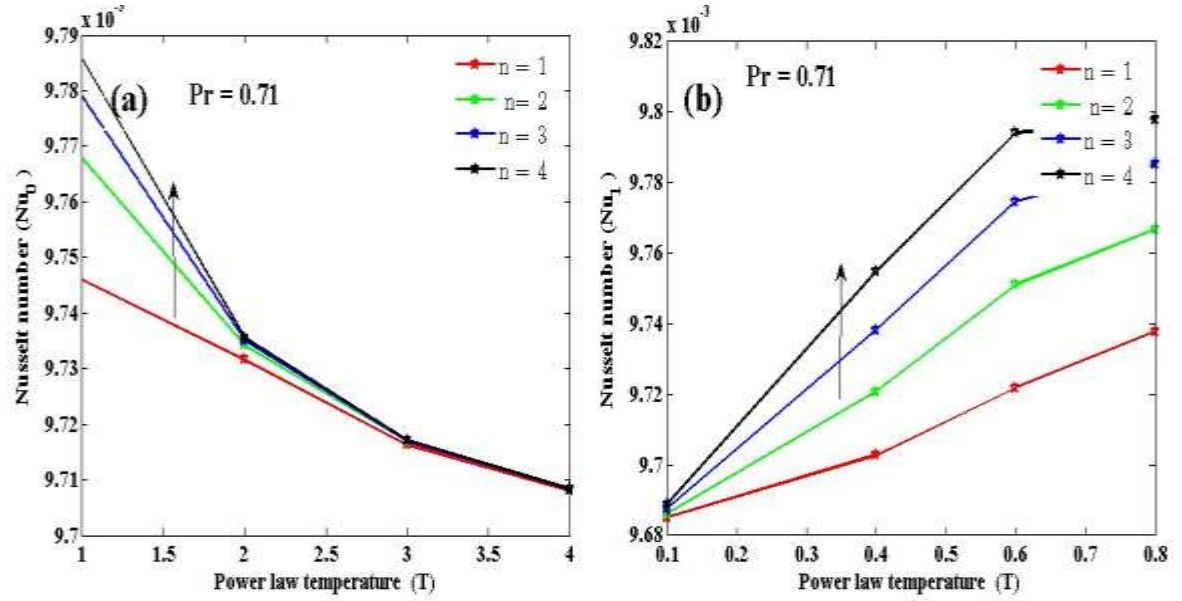

Figure 18: influence of power law temperature and index on Nusselt number

Figures 15 (a) and (b) show the impact of Biot number and time on Nusselt number at heated and cooled walls (Pr = 0.71) respectively. Nusselt number decline with rise in Biot number and time at heated wall, while the heat transfer rises with increase in Biot number and time at cooled wall. Figures 16 (a) and (b) indicate the effect of thermal radiation parameter R and temperature difference C_T on Nusselt number (Pr = 0.71) at convective and non-convective walls respectively. It is observed that, at convective wall Nusselt number falls with rise in R and C_T while it rises with rise in R and C_T at non-convective wall. Figure 17 (Pr = 7.0) shows the same properties with that of figure 16. Figures 18 (a) and (b) reveal the influence of power law temperature and index on heat transfer at heated and cooled plates respectively.

Heat transfer increases with rise in index at both plates but the heat transfer decline with increase in power law temperature at heated plate increase on the cooled plate.

CONCLUSION

Steady/unsteady heat transfers flow of viscid, non-compressible and electrically conduction fluid (MHD) within two vertical channel with power temperature dependent heat flux due to convective boundary condition has been examined. The possessions of non-dimensional controlling parameters on velocity $u(y,t)$, temperature $\theta(y,t)$, skin friction $\tau_{0,1}$ and Nusselt number $Nu_{0,1}$ were explored and reported using line graph. The finding revealed that:

(i) The analytical and numerical solution agreed (merged) for large value of time

(ii) Convective boundary condition significantly affect the velocity, temperature, skin friction and Nusselt number of the flow.

(iii) Upturn in M leads to decline in velocity and skin friction while upsurge in R and C_T lead to increase in velocity, temperature and skin friction

(iv) Power law temperature and index significantly influenced the heat transfer and skin friction of the flow

(v) Water takes almost twice the time take by air to reach steady state.

(vi) Increasing effects of time t, radiation parameter, temperature difference and Biot number lead to decrease in Nusselt number at y = 0 with reverse effect at y = 1.

Contribution to knowledge

The contribution of this research scientifically is the particular way it combines analytical and numerical methods to investigate time dependent Magneto hydrodynamic free convective heat transfer flow in vertical non-porous and porous channels in the presence of nonlinear thermal radiation parameter with other non-dimensional controlling parameters subjected to convective boundary condition and slip velocity on the extended models. Convectivity of the flow channel boundary is found to be neglected by many researchers because non-convective boundary condition is obstinate. The content of this research particularly studied;

i. To improve the existing model by cooperating non-linear convective boundary condition, power law temperature, thermal radiation , and variable viscosity.

ii. To find out analytical and numerical solutions of steady/transient MHD free convective flow in the present of power law temperature and index parameter within vertical parallel plates.

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REFERENCES

Asia Y, Kashif A & Muhammad A (2020) **Study of** Heat and Mass Transfer in MHD Flow of Micropolar Fluid over a Curved Stretching Sheet**.** *Scientific Reports* **volume 10**, Article number: 4581

Bashiru A., Isah B. Y., Seini I. Y , and Hamza M. M. Upshot of Slip Velocity and Thermal Radiation on Magneto hydrodynamic transient fluid flow through vertical walls. *International Journal of Theoretical and Applied Mathematics*. Vol 8:3. pp 65-77

B. Y Isah, B. Abdullahi and I. Y. Seini (2022). On transient MHD heat transfer within a radiative porous channel due to convective boundary conditions. Willey publisher vol. 51:6, pp 5140-5158

Falana A (2021) Velocity Slip Effect on MHD Power-Law Fluid over a Moving Surface with Heat Generation, Viscous Dissipation and Thermal Radiation. Vol 20 (10) pp**:** 103-112

Nora M. A. and Sivasankaran S. (2024) Entropy Generation and Thermal Radiation Impact on Magneto-Convective Flow of Heat-Generating Hybrid Nano-Liquid in a Non-Darcy Porous Medium with Non-Uniform Heat Flux. Journal computation. Vol 12(3)

Nor A.A., Anuar I. and Loan P (2020) Magnetohydrodynamics Flow and Heat Transfer Induced by a Shrinking Sheet *Mathematics 8*(7), 1175

Panel J. K M., Subhasisa R. and Sukanta K. D. (2021) Relative importance of temperature-dependent properties in non-Newtonian natural convection around curved surfaces. International Communications in Heat and Mass Transfer. Vol 124, 105263.

Ramana M.V. M, Srinivasa R. R and Anand J.R. (2015) Heat and Mass Transfer Effects on MHD Natural Convective Flow Past an Infinite Vertical Porous Plate with Thermal Radiation and Hall Current. Procedia Engineering 127 DOI: 10.1016/j.proeng.2015.11.491

Rashad, A.M. (2017) Impact of thermal radiation on MHD slip flow of a Ferro fluid over a non-isothermal wedge. J. Magn. Magn. Mater. **Vol.** 422, pp 25–31

Sandhya A. Ramana G. V R, Deekshitulu V. S.R.G. (2020) Heat and mass transfer effects on MHD flow past an inclined porous plate in the presence of chemical reaction. International Journal of Applied Mechanics pp 86-102

Shankar B.G., Dharmendar Y. R., S.R. Mishra S.R., Ijaz M. K, Kamel G, Ahmed M. G. (2022) Thermal radiation impact on magneto-hydrodynamic heat transfer micro polar fluid flow over a vertical moving porous plate: A finite difference approach. Journal of the Indian Chemical Society. Vol**.** 99(8), 100618

Syed A. G. A, Ali H., Hanen K, Abdullah A, Sayed M. E, and Ahmed M. G (2023) Effect of thermal radiation on convective heat transfer in MHD boundary layer Carreau fluid with chemical reaction Sci Rep. 2023; 13: 4117.

Varun R. S.K, Ioannis E. S. Sowmya G. and Amal A. (2023) Iterative Solutions for the Nonlinear Heat Transfer Equation of a Convective-Radiative Annular Fin with Power Law Temperature-Dependent Thermal Properties. Journal of symmetry vol. 15 (6)