International Journal of Innovative Agriculture & Biology Research 11(1):32-40, Jan.-Mar., 2023

 www.seahipaj.org **ISSN: 2354- 2934**

Diameter Distribution of *Cedrela odorata* Using Gamma and Beta Distribution Functions in Ijare Plantation, Ogun State, Nigeria

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ABSTRACT
Diameter class models enable planning of various uses and provide information about stand structure. The objective of this study was to compare the effectiveness of both gamma and beta distribution functions in describing the diameter distribution of *Cedrela odorata* stands in Ijare, Ogun State. The sampling design used was stratified random sampling, where sampling units differ in terms of age constituting the strata, which are ages, 14 and 15 years. Sample plots of 20mx 20m were randomly selected from each stratum and complete enumeration done for all trees in the sample plots selected. The variable measured was diameter at breast height over bark (Dbh) for all trees. The data was analyzed for beta and gamma distribution functions using maximum likelihood approach. The comparison was based on Kolmogorov-Simonov (K-S), bias, means absolute error and root mean square error. The values for K-S was 0.451, RMSE = 0.027, MAE = 0.044 and Bias = 0.05 for beta distribution function respectively while that of gamma distribution were 1.521, 1.878, 1.487 and 1.417 respectively. Based on the parameter estimation method, the beta distribution gave a better result than the gamma distribution function. Beta distribution function is therefore recommended for diameter distribution of *Cedrela odorata* in the study area.
Keywords: Diameter at breast height, diameter distribution, *Cedrela odorata*, Gamma and Beta distributions

INTRODUCTION

Effective and efficient forest management is only possible when reliable information about the present and future forest condition is available. Research has revealed that modeling growth has been an intrinsic part of forest research for several years and remain an area of important and active research (Vanclay, 1994; Porte and Bartelink, 2002 and Ureigho and Akpobome, 2018). The ability to predict growth and yield in the forest is an essential requirement for effective management of the forest resources (Ureigho and Osho, 2017). In natural forest stands, individual tree height relationship and prediction models are often complicated by selective harvesting and other disturbances, and by changing stand dynamics (Oliver and Larson, 1990). A knowledge of stand growth and increment models is one of the most important requirements for intelligent forest management. Estimate of stand growth are needed to decide the health of the forest, the volume of materials that can be harvested without violating the sustainability of the forest. Forest growth modelling is no exception; models must evolve to remain relevant (Vanclay, 2003).

Stand growth is established through modelling or by sampling the actual tree growth to predict potential yields in the future (Ureigho, 2021). Thus increasing demand for better forest management has created new challenges for modellers to provide growth and yield models to deal with tree growth and predict future yield in stands comprising many species and a wide range of the sizes (Vanclay, 2006,Ureigho and Nkpaji 2020).

The quality of forest management plans hinges on the quality and reliability of growth and yield predictions. Traditionally, growth and yield models are developed from large amounts of historical data using statistical techniques (Weiskittel *et al*., 2011). The models usually require simple inputs and predict with relatively low bias at regional scales (Monserud, 2003). The models have the capability to produce information from individual tree; stand and forest levels and such information can be sourced from internet according to Ureigho *et al*., (2005). However, these models are empirical in nature and as a result are applicable only to stand growing conditions similar to the conditions for which they were developed (Monserud, 2003, Mieble *et al*., 2009).

Gamma and Beta distribution functions describe the current and future state of the forest stand. These functions are beneficial and essential instrument in the area of forest management. Gamma and Beta are sub-models under statistical distribution function (Podlaski, 2006). Beta distribution is very flexible and is commonly used to represent variability over a fixed range. Beta is used to describe empirical data and predict the random behaviour of percentages and fractions as the range of outcomes is typically between 0 and 1. The beta is used as a conjugate prior distribution for binomial probabilities in Bayesian statistics. Gamma distribution applies to a wide range of physical quantities and is related to other distribution like Lognormal, Exponential, Pascal, Erlang, Poisson and Chi- square. Gamma distribution is used for inventory control and it is applicable to other distributions like economic control and Insurance risk theory. Statistical distribution has the ability to prepare the growth models, examination of dispersion and estimation of its figures in terms of management aspects (Podlaski, 2006), (Namiranian, 2007**).** Gamma distribution is a two-parameter family of continuous probability distribution while the beta distribution is a family of continuous probability distributions defined on the interval (0, 1). Forest managers are interested in estimating the number of trees of different diameter classes in a stand because the size of the diameter determines the industrial use of the wood and thus the price of the different products (Ureigho, 2018 and Ureigho 2013). Diameter distributions also provide information about stand structure, age structure, stand stability e.t.c and enable the planning of silvicultural treatments. Therefore, there is need for wide variety of models of varying degree of complexity for the management of natural forest and plantation. Diameter at breast height (Dbh) of forest trees is an essential variable in determining the basal area and more importantly the growth of the forest. It is the easiest measured variable, which can be used to predict the growth of the forest estate.

Cedrala odorata is in the family of *Meliaceae*. It has great potential in reforestation because of characteristics like fast growth, easy management in the nursery, adaptability to different soils, climatic conditions and high growth rate in agroforestry systems (Navarro, 2002). The wood is the traditional choice for making classical guitars (Romanillos, 2014).

The main objective of this study was to compare the effectiveness of both gamma and beta distribution functions in describing the diameter distribution of *Cedrela odorata* stands in Ijare, Ogun State.

MATERIALS AND METHODS

The study area is Ijare *Cedrela odorata* plantation. It is located in Ijebu- North Local Government Area of Ogun state at an altitude between Latitude 6° 49'N, 9°98'E and Longitude $3^0$55'N, $2^0$32'E of the Greenwich meridian. The vegetation is a tropical rainforest with mean annual rainfall, which varies from 1800-2000 mm. Stratified random sampling technique, was used in the collection of data from the plantation. The different strata consisting of ages 9 and 10 were divided into different sample plots of 20x20m and complete enumeration of all trees were carried out in each sample plots selected. Diameter at breast height over bark (dbh) was measured at a standard position of 1.3m above the ground with the aid of Diameter girth tape for all trees in the sample plots selected.

Beta Distribution. This is the distribution when a random variable X has a density function given by;

$$f(x)=\frac{\sigma\ (p+q)}{\sigma\ (p)\ \alpha\ (q)} - X^{p-1}\ (1-X)^{q-1}$$

where,

0< X <1, P>O, Q>0 p and q are the beta constants to be estimated and X is the diameter.

The gamma and beta distribution function have their density function highly flexible in shapes and can therefore be adapted for growth studies.

Gamma distribution

A continuous random variable X has a gamma distribution when its probability density function is given as;

$$f(x) = \frac{A^{\beta}\ X^{\beta-1}}{\sigma(\beta)}\ expe^{-Ax}$$

Where A, β and σ are gamma constants to be estimated and X is the diameter. This model is noted for generating positively skewed curve.

After the parameters of the distributions have been, estimated using the methods of moment, parameters were fitted to the distribution functions. This was used to obtain the class probabilities (*P*i) and subsequently used to compute the diameter-class frequencies for each plot.

Predicted Number of tree per class (Ni) = N *x* Pi

Where: Ni = estimated number of trees per class, N = number of trees per ha and Pi = class probability.

The following goodness-of-fit indices were used to test the ability of the Beta and Gamma distributions to the diameter distribution of the species in Ijare plantation stand.

Kolmogorov-Smirnov (KS) test: This was used to compare the cumulative estimated frequency with the observed frequency. The most striking difference between the two distributions was the *D*n statistic value of the

KS test: $Dn = Supx|\ F(x_i) - F_o(x_i)|$

Where: sup*x* is the supremum value for *x*

$$D_n = max\{max_{1\le i\le n_i}[F_n(x_i) - F_0(x_i)], max_{1\le i\le n_i}[F_o(x_i) - F_n(x_{i-1})]\}$$

Fn(xi) is the cumulative frequency distribution observed for the sample *xi* (*i* = 1, 2, n)

F0(xi) is the probability of the theoretical cumulative frequency distribution.

RESULTS AND DISCUSSION

The results of the analysis of diameter at breast height and diameter distributions are shown below. This was based on the comparison of their probability density functions. Table 1 showed the frequency distribution of DBH. The trees were distributed into different diameter classes. It was observed that more of the trees fell in DBH class between 40.00 – 59.99 at both ages 13 and 15. The result is further explained by Figure 1 below, which shows the curve of the Diameter classes against the frequency of each class.

Table 1: Frequency Distribution of Diameter at Breast height according to the age series.

DBH Class	Age 9	Age 10
0 - 19.99	0	0
20.00 - 39.99	15	46
40.00 - 59.99	30	83
60.00 - 79.99	12	33
80.00 and above	3	4
Total	60	166

Source: Author's Field Survey

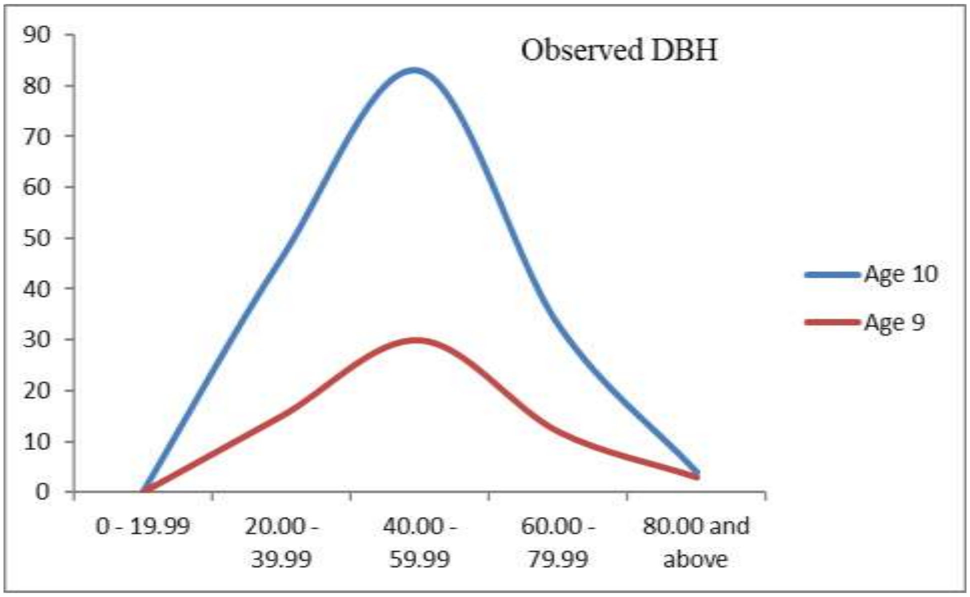

Figure 1: Curve of the Observed DBH

Beta and Gamma Distribution Model

Maximum Likelihood Method was used to estimate parameters for Beta and gamma distribution functions. The Maximum Likelihood estimation method was preferred to other estimation methods such as Moment Method because of its reliability and its ability to predict more accurately. As shown in Table 2, the estimated scale parameter (a) is 633.559 while the shape parameter is 620.834. On the other hand, the scale parameter for Gamma distribution is 12.019 while the shape parameter is 4.142

Table 2: Model Parameters of Beta and Gamma distribution functions

Model	Parameters	Values
Beta	Scale (a)	633.559
	Shape (b)	620.834
Gamma	Scale (a)	12.019
	Shape (b)	4.142

Diameter Classes as Distributed by Beta Distribution

The result shown in Table 3 is Diameter at breast height fitted into Beta Distribution using Beta Probability Density Function, after the shape and scale parameters have being estimated using maximum likelihood estimator. Majority of the trees fell in diameter class 20.00 – 39.99 in ages 9 and 10. This is in line with Osamionayi *et al*., (2020) who observed that majority of the trees in his study fell within this range, which implies that regeneration of the trees was high. Figure 2 below further depicts the results of the analysis.

Table 3: Beta Distribution of DBH into Diameter classes

Beta Dist. Diameter Class	**Age**		**Total**
	Age 10	Age 9	
0 - 19.99	7	4	11
20.00 - 39.99	95	34	129
40.00 - 59.99	64	22	86
60.00 - 79.99	0	0	0
80.00 and above	0	0	0
Total	166	60	226

Source: Author's Field Survey, 2018

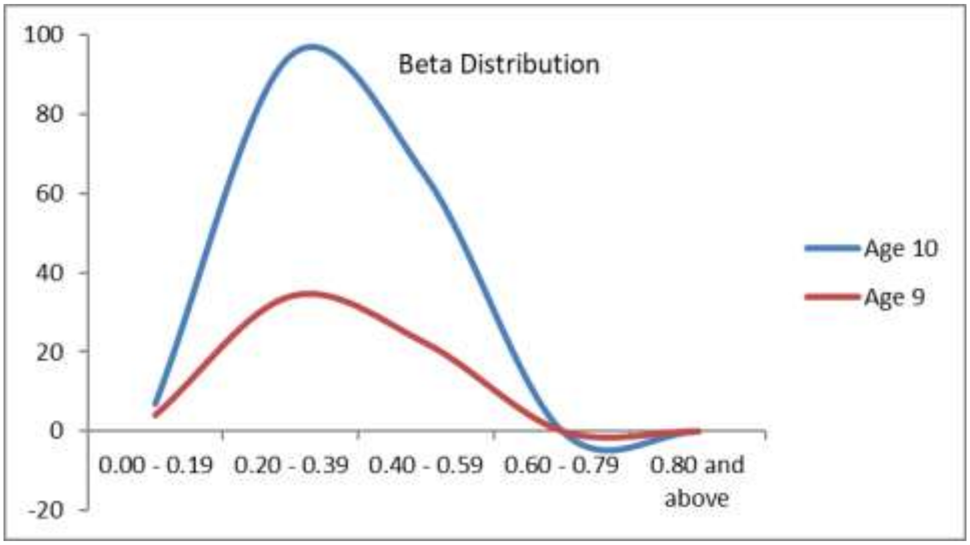

Figure 4. Curve of Beta Distribution

Diameter Classes as Distributed by Gamma Distribution

The results derived on fitting DBH into Gamma Distribution according to the different age of trees is presented in Table 4. Majority of trees fell in diameter class 0 – 19.99 fell in age 10 while majority fell in 40.00 – 59.99 in age 9.

Table 4. Gamma Distribution of DBH into Diameter Classes

Gamma Dist. Diameter Class	**Age**		**Total**
	Age 10	Age 9	
0 - 19.99	40	10	50
20.00 - 39.99	25	12	37
40.00 - 59.99	34	14	48
60.00 - 79.99	34	12	46
80.00 and above	33	12	45
Total	166	60	226

Source: Author's Field Survey

Comparison of the Results of Beta Distribution and Gamma Distribution

The results obtained from the comparison of the diameter class distribution obtained from data fitted into Beta and Gamma distributions are shown in table 6. The result showed that Beta distribution indicates that 129 of the trees falls between diameter class 20 to 39.9 while according to the result of Gamma Distribution, more of the trees have lower diameter. The results of this comparison is further presented in Figure 5 where the diameter classes and their frequencies were plotted against each other

Table 5: Comparison of DBH Observed and DBH predicted using Beta and gamma Distribution

Diameter Class	DBH	Beta	Gamma
0 - 19.99	0	11	50
20.00 - 39.99	61	129	37
40.00 - 59.99	113	86	48
60.00 - 79.99	45	0	46
80.00 and above	7	0	45
Total	226	226	226

Source: Author's Field Survey

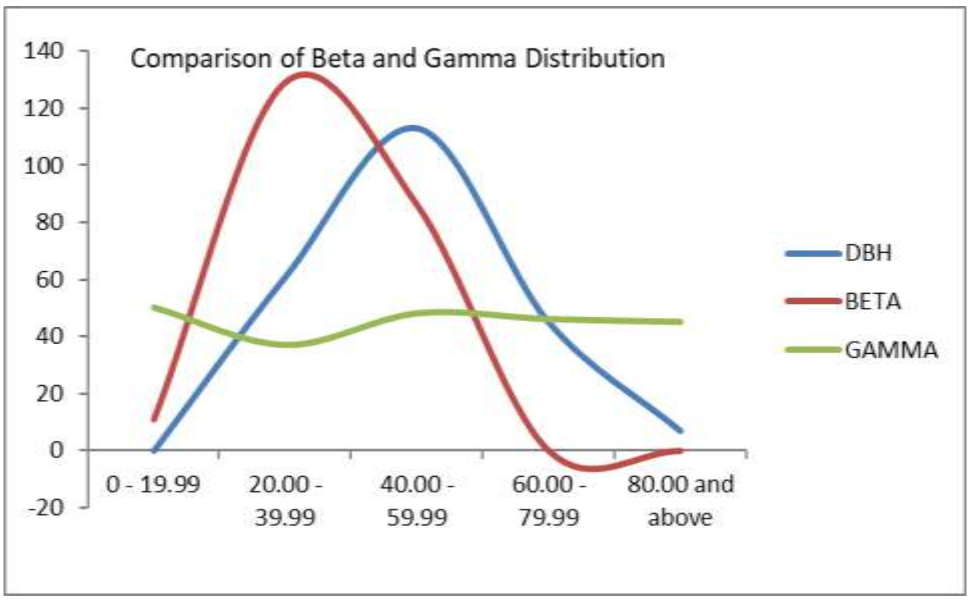

Figure 5. Curves of the Comparison of Beta and Gamma Distribution

The results of analyses carried out using estimated parameter further revealed that the Beta distribution was better for all the criteria used. Clear differences in the performance of the distribution functions could be identified (Table 6). The overall ranking in terms of mean values of bias, Mean Absolute Error (MAE), Root Mean Square Error (RMSE), and mean value of the Kolmogorov-Smirnov (K-S) statistic summarizes the overall accuracy of the distribution functions as comparison criteria. The results showed that the Beta distribution had the smallest K-S mean value of 0.451, smallest RMSE, MAE and bias of 0.027, 0.044, and 0.056, respectively. This proved the superiority of the Beta distribution function over the Gamma distribution in characterizing the diameter at breast height. The Gamma distribution whose K-S statistic (1.521) and bias (1.417) revealed poor fits.

Table 6: Goodness of fit test for Beta and Gamma distributions

Comparison Criteria	Beta	Gamma
Bias	0.056	1.417
RMSE	0.027	1.878
MAE	0.044	1.487
K-S	0.451	1.521

DISCUSSION

The classification of the tree diameter of the stands of Ijare plantation was the hallmark of this study. The effectiveness of Beta and Gamma distributions for diameter classification was compared in pursuant of the best distribution function that could determine the structure of the forest stand. The Maximum Likelihood Estimation Method was used in the parameter estimation of the distribution functions. The assessment of the classification ability of the distribution functions based on the goodness of fit statistics (i.e. K-S, bias, MAE, and RMSE) revealed that the Beta distribution provided the best fit to the dataset for the ten (10) plots considered. The relative flexibility of the Beta distribution could have influenced it performance. Attributed to this could also be because of the effects of the some of the assumptions made in this study i.e. location parameter is assumed not to be equal to 0. This study is in agreement with the findings of Zhang *et al.* (2003) who opined that the results of the fitted distribution depend on the underlying assumptions used in the fitting process, and concluded that the assumption concerning some parameters could have been more refine. This result also conformed to Zheng and Zhous (2010) who modeled diameter distribution of trees in natural stands managed on polycyclic cutting system using beta, Gamma, negative Exponential and Weibull distributions. Though they were of the opinion that Weibull distribution model fitted better than others regarding the structure of diameter distribution in natural forests managed on polycyclic cutting system, Beta distribution is preferred to Gamma distribution. The finding agreed with Fallahchai, (2000) who investigated the structure of natural beech stands using different statistical distributions; found that the Beta distribution was more appropriate for the diameter characterization of the forest stands. However, this is not in line with Bullock and Boone (2007) who studied diameter distributions of loblolly pine trees, observed that sometimes none of the distributions may fit the dataset and concluded that in such situation Bayesian model, averaging distribution could be used.

Furthermore, this finding agreed with Gorgoso *et al.,* (2012) who compared the accuracy of the Weibull distribution, Johnson SB and beta distributions for describing the diameter distributions in even-aged stands of three pines species; found that the Johnson SB and Beta distribution were more superior than the Weibull distribution for two of the species.

The fit provided by the 2-parameter Gamma distribution for this study showed that the 2-parameter Gamma was not adequate for even-aged stands. This study agreed with Eslami *et al.,* (2011) who investigated the structure and distribution of diameter classes in beech forest and reported that the Gamma distribution did not fit the dataset, as such recommended the beta distribution model for the natural stand. However, this finding is not in line with Mohammad *et al.,* (2009) and Ureigho and Osho (2017), who studied the diameter distribution in uneven-aged forest, stand and observed that the Gamma distribution provided the best fit for the forest stand. From the foregoing discussion, it seems that fitting the same dataset with 3-parameter Gamma distribution may provide a good fit, since the number of parameters affect distribution performance.

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CONCLUSION

Growth models are very useful in forestry for management and decision-making. This study was carried out to estimate the best model with best fit for diameter distribution in Ijare plantation in Ijebu-Ode North local government area of Ogun state. The beta distribution function gave the best fit for this study. This study has provided some baseline information on the diameter distribution of the plantation stands of Ijare plantation. The comparison on the effectiveness of the Beta and Gamma distributions for classifying the tree diameter of the reserve has been made. The results from the goodness-of fit statistics i.e. Kolmogorov-Smirnov (K-S), bias, mean absolute error (MAE), and root mean square error (RMSE) indicated that the Beta distribution gave fit of the data. In the case of the Gamma distribution, poor fits and inconsistency were observed for the entire plots used for this study, which implies that the Gamma distribution cannot determine the true structure of the tree stands.

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