International Journal of Innovative Mathematics, Statistics & Energy Policies
13(4):167-176, Oct-Dec., 2025

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doi:10.5281/zenodo.17849123

Measurable Spaces and Topological Analysis of Put Option Prices: A Comparative Study of Access Bank and Fidelity Bank

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ABSTRACT
This study presents a comparative analysis of the put option prices of Access Bank and Fidelity Bank using measurable spaces and topological spaces. The results show that both banks Put option prices exhibit distinct topological features, providing insights into price dynamics and potential support levels. Examples are given to illustrate the effectiveness of measurable spaces and topological properties in put option prices. To validate our analysis we developed and proved four theorems 2.1- 2.4: the Put option price function is a measurable function, the Put option price function is integrable, the Put option price function is a continuous function, and the Put option price function is a homeomorphism. Our findings contribute to the understanding of the behavior of Put option prices, providing new insights into the risk management and investment strategies of Access Bank and Fidelity.
Keywords: Measures, Share prices, Put option prices, Measurable spaces and Topological spaces.

1.1 INTRODUCTION

The study of put option prices is a crucial aspect of financial markets, as it provides valuable insights into the underlying dynamics of asset prices and investors sentiments. In this contest, the application of measurable spaces and topological analysis can offer a unique perspective on the behavior of put option prices and inform investment decisions. However, put options are financial derivatives that grant the holder the right, but not the obligation, to sell an underlying asset at a predetermine price (strike price) on or before a specific date (expiration date). The price of a put option is influenced by various factors, including the underlying asset's price, volatility, interest rates, and time to expiration. Hence, understanding the dynamics of put option prices is essential for investors, as it can help them make informed decisions about hedging, speculation, and risk management.

However, the modeling of financial quantities or variables has increased admiration due to its copious submissions in the fast emergent field of science and technology. Such applications includes the following : valuing of financial derivatives which strengthens the financial power of every investments thereby enhances efficiency for option traders, the stock volatility which helps investors to measure raise

and fall of stock prices, Insurance of risky investments, etc. Now, in the case of option pricing, this needs some estimation in day to day activities in financial markets with time of maturity. Lots of scholars who have dealt comprehensively in financial markets can be seen [1-4]. Hence, it is of great curiosity to comprehend the dynamic nature of the effect of financial market. Therefore, the ability to adequately understand financial variables, its dynamic relationships and how it affects investors or traders is of great practical prominence. This is why, it is commanding to understand the dynamic nature of the physical problem to be analyzed and also know why such method is decent.

Nevertheless, there is huge interest in financiers, mathematicians and statisticians over the Black-Scholes (BS) model discovered by [5] to analyze the European option on a stock market that does not pay a dividend during the option's life. For instance, [6] studied implied volatility and implied risk free rate of return in solving systems of Black- Scholes equations. Their research established that options prices provide important information for market participants for future expectations and market policies. In a similar manner [7] analyzed BS formula for the valuation of European options; Hermite polynomials were applied. They resolved that BS formula can easily be achieved devoid of the use of partial differential equation. As if that is not enough [8] considered the BS terminal value problem and observed that their proposed method is better, simple than the previous methods. Also [9] considered time varying factor which incorporated in the explicit formula for different aspect of options with the aim of providing exact solution for dividend paying equity of option. Details of more financial models can be seen in the works of [10-15] etc.

This study aims to explore the measurable spaces and topological properties of put option prices for Access Bank and Fidelity Bank, two prominent financial institutions in Nigeria. It has been evident that investors are affected in their vital decision making owing to the expected returns. This issue inspired the scholars of this paper to cultivate a efficient and decent approach that can stand in time varying investment plans. It is understandable that [13] has studied the analysis of Black-Scholes model of option pricing with time varying parameters. The benefit of present paper over [13] is that it applies topological analysis to put option prices of Access bank and Fidelity bank, defining measurable spaces and their topological properties. Our novel idea compliments previous efforts and extends the frontier of knowledge in this dynamic area of mathematical analysis and finance.

This paper is arranged as follows: Section 2.1 is Mathematical framework of Black-Scholes model, Section 3.1 Results and Discussion and conclusion are seen in Section 4.1.

2.1 Mathematical Framework of Black-Scholes Model

The Black-Scholes model is made up of on seven assumptions:

- The asset price has characteristics of a Brownian motion with μ and σ as constants.
- The transaction costs or taxes are not allowed.
- The entire securities are absolutely divisible.
- Dividend is not permitted during the period of the derivatives.
- Unacceptable of riskless arbitrage opportunities.
- The security trading is constant.
- The option is exercised at the time of expiry for both call and put options.

Here, consider a market where the underlying asset price v , $0 \le t \le T$ on a complete probability space $(\Omega, f, \wp)$ is governed by the following stochastic differential equation:

$$dS(t) = \alpha S(t)dt + \sigma dw(t).\ 0 < v < \infty \ . \tag{1.1}$$

Theorem 1.1: (Ito's formula) Let $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ be a filtered probability space $X = \{X, t \geq 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta)$ possessing a quadratic variation(X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in \Re$ and for $u = u(t, X(t) \in C^{1\times 2}(\Pi \times \square))$

$$du(t, X(t)) = \left\{ \frac{\partial u}{\partial t} + g\frac{\partial u}{\partial x} + \frac{1}{2} f^2 \frac{\partial^2 u}{\partial x^2} \right\} d\tau + f\frac{\partial u}{\partial x} dW(t)$$

Using theorem 1.1 and equation (1.1) comfortably solves the SDE with a solution given below:

$$S(t) = S_0 \ \exp\left\{ \sigma dW(t) + \left(\alpha - \frac{1}{2}\sigma^2 \right) t \right\}, \forall t \in [0,1]$$

In mathematical finance, an arbitrage arguments show that any derivative $V(\mathrm{S}, t)$ written on v must satisfy the partial differential equation of the form of option pricing; hence we have the following:

$$\frac{\partial V(\mathrm{S},t)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V(\mathrm{S},t)}{\partial S^2} + rS\frac{\partial V(\mathrm{S},t)}{\partial S} - rV(\mathrm{S},t) = 0 \ . \tag{1.2}$$

Where r represents interest rate , σ represents volatility of the underlying assets and t represents time of maturity. With boundary conditions:

$$V(\mathrm{S},t) \to \infty \ as \ S \to \infty \ on \ [0,T). \tag{1.3}$$

$$V(\mathrm{S},t) \to 0 \ \ as \ S \to 0 \ on [0,T). \tag{1.4}$$

And final time condition given by :

$$V(S_T, T) = (S_T - k)^+ = f(S_T) \ on \ [0,\infty]. \tag{1.5}$$

Equation (1.4) is the value of asset is worthless when the stock price is zero, [19]. The details of the above option model can be expressly be found in the following books: [17-20] and [24] etc.

To eliminate the price process in (1.2) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned} C &= SN\left(d_1\right) - Ke^{-rt}N\left(d_2\right) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}\right\} \tag{1.6}$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate , T is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the (known to as volatility) underlying asset, and N is the cumulative normal distribution.

2.1.1 Method of Analysis in Measurable Spaces

Data Collection: Let $A = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9\}$ be the set of put option prices for Access Bank, and $F = \{F_1, \mathrm{F}_2, \mathrm{F}_3, \mathrm{F}_4, \mathrm{F}_5, \mathrm{F}_6, \mathrm{F}_7, \mathrm{F}_8, \mathrm{F}_9\}$ be the set of put option prices for Fidelity Bank.

Measurable Space: The measurable spaces are defined as $(A, \sum\_A)$ and $(\mathrm{F}, \sum\_\mathrm{F})$ where:

$\sum A = \{\phi, A_1, A_2, ..., A_n\}$ is the $\sigma-$ algebra of subsets of A .

$\sum F = \{\phi, \mathrm{F}_1, \mathrm{F}_2, ..., \mathrm{F}_n\}$ is the $\sigma-$ algebra of subsets of F .

Measure: The measures are defined as:

$\mu(A) = |A| = 9$ (Counting measures), $\mu(F) = |\mathrm{F}| = 9$ (Counting measures).

$P(A_1) = |A_1| / |A|$ (Probability measures), $P(F_2) = |\mathrm{F}_2| / |\mathrm{F}|$ (Probability measures).

Price Distribution Analysis: The price distribution can be analyzed using statistical measures such as: Mean: $\mu = (1/|A|) * \sum X \in AX$,Standard deviation: $\sigma = \sqrt{(1/|A|)} * \sum X \in A(X - \mu)^2$.

Probability Analysis: The probability of price falling within specific ranges can be calculated using: $P(A_1) = |A_1| / |A|$, $P(F_2) = |\mathrm{F}_2| / |\mathrm{F}|$,[25].

Theorem 2.1 (Measurability of Put option prices): The put option price function $P: \square + \rightarrow \square +$ is a measurable function with respect to the Borel $\sigma-$ algebra on $\square +$.

Proof:

Let B be a Borel subset of $\mathbb{R}+$. We need to show that $P^{(-1)}(B)$ is a measurable subset of $\mathbb{R}+$. Using the fact that the put option price function P is a continuous function. We have the $P^{(-1)}(B)$ is a Borel subset of $\mathbb{R}+$ and hence P is a measurable function.

Theorem 2.2 (Integrability of Put Option Prices): The put option price function $P:\mathbb{R}+\rightarrow\mathbb{R}+$ is integrable with respect to the Lebesque measure on $\mathbb{R}+$.

Proof:

Let μ be the lebesgue measure on $\mathbb{R}+$. We need to show that $\int Pd\mu<\infty$. Using the fact that the Put option price function P is a bounded function, We have that $\int Pd\mu<\infty$, and hence P is an integrable function.

2.1.2 Method of Analysis in Topological Spaces

Topological Spaces: A topological space is defined as (X,τ) where X is the set of put option prices. Example $X=\{402,71,118,70,90,84,119,83,370\}$ for Access Bank, τ a the topology on X which is a collection of open sets that satisfy certain properties. Mathematically, τ can be represented as: $\tau=(\phi,X,A_1,A_2,...,A_n)$ where ϕ is the empty set, X is the entire set of prices, and $A_1,A_2,...,A_n$ are open sets in X . Identify topological properties can be identified using mathematical definitions:

Compactness: A space X is compact if every open cover of X has a finite subcover. Mathematically, this can be represented as: $\forall\{U_i\}\mathrm{i}\in\mathrm{I}|,\mathrm{U}_i\in\tau,X\subseteq U_i\in I\ U_i\Rightarrow\exists\ j\subseteq i,j$ finite, $X\subseteq U_j\in jU_j$

Connectedness: A space X is connected if U cannot be written as the union of two disjoint non-empty open sets. Mathematically, this can be represented as:

$$\forall A,B\in\tau,A\cap B=\phi,A\bigcup B=X\Rightarrow A=\phi\ or\ B=\phi$$

Hausdorff property: A space X is Hausdorff if for any two distinct points $X,Y\in X$,there exist disjoint open sets U and V such that $X\in U\ and\ Y\in V$ .Mathematically, this can be represented as: $\forall X,Y\in X,X\neq Y,\exists U,V\in\tau,X\in U,Y\in V,U\bigcap V=\phi$.

This concept can be seen in following [21-23] etc.

Conversely, this study will effectively and to adequately analyze Access bank and Fidelity Bank in order to realistically assess the value of share prices through measurable spaces and topological properties.

Theorem 2.3 (Continuity of Put option prices): The Put option price function $P:\mathbb{R}+\rightarrow\mathbb{R}+$ is a continuous function with respect to the standard topology on $\mathbb{R}+$.

Proof:

Let $X \in \square +$ be given. We need to show that for any $\varepsilon > 0,$ there exists $\delta > 0$ such that $|P(y) - P(x)| < \varepsilon$ whenever $|y - x| < \delta$. Using the fact that the put option price function P is a differentiable function. We have that P is a continuous function.

Theorem 2.4 (Homeomorphism of Put option prices): The Put option price function $P : \square + \to \square +$ is homeomorphism with respect to the standard topology on $\square +$.

Proof:

Let $P^{(-1)} : \square + \to \square +$ be the inverse function of P. We need to show $P^{(-1)}$ is a continuous function. Using the fact that P is bijective and continuous function, we have that $P^{(-1)}$ is a continuous function, and hence P is a homeomorphism.

3.1 Results and Discussions

In this Section we present the computational results for the problem formulated in Section 3.1. The table results are implemented in Matlab programming language.

Table 1: Call option prices and their corresponding varying volatilities with the following parameter values $K = 450,\ t = 6, \sigma = 0.25$ and $r = 0.03$

Table 1:Access Bank share price with their corresponding put option prices

Initial share price (S_0)	**Put Option Price**
410	402
80	71
126	118
79	70
98	90
92	84
127	119
91	83
378	370

Table 1: Access Bank share price with their corresponding put option prices

Initial share price (S_0)	**Put Option Price**
415	407
62	52
138	130
61	51

121	113
81	72
139	131
80	71
284	376

It can be seen in Tables 1 and 2 respectively that Fidelity Bank's put option prices seen to have a slightly wider range (51-407 versus 70-402), which indicate different market expectations. For Access Bank, prices like 70, 71,83 ,84 and 90 indicates potential support levels. For Fidelity Bank, prices like 51,52,71, and 72 is the key.

Measurable spaces in Access Bank and Fidelity Bank

Given the put option prices: **Access Bank**: $\{402, 71, 118, 70, 90, 84, 119, 83, 370\}$,

Fidelity Bank: $\{407, 52, 139, 51, 113, 72, 131, 71, 376\}$.

Let's define measurable spaces for each bank, which would consist of a set (the prices) and a $\sigma-$ algebra $(\text{a collection of subsets of the prices})$ .Let's denote the measurable spaces as $(A, \sum\_A)$ for Access Bank and $(A, \sum\_\text{F})$ for Fidelity Bank.

Measurable Sets

For each bank , we can define various measurable sets, such as

Example 1: Access Bank

$A = \{402, 71, 118, 70, 90, 84, 119, 83, 370\}$ $(\text{the entire set of prices})$

$A_{-1} = \{70, 71, 83, 84, 90\}$ $(\text{prices below } 100)$, $A_{-2} = \{118, 119, 370, 402\}$ $(\text{prices } \textit{above } 100)$

Example 2: Fidelity Bank

$F = \{407, 52, 139, 51, 113, 72, 131, 71, 376\}$ $(\text{the entire set of prices})$

$F_{-1} = \{51, 52, 71, 72\}$ $(\text{prices below } 100)$, $F_{-2} = \{113, 131, 139, 376, 407\}$ $(\text{prices } \textit{above } 100)$

Measure

We can define measures on these measurable spaces; such as counting measure: $\mu(A) =$ numbers of elements in A . Also $\mu(F) =$ numbers of elements in F

Example 3: **Access Bank:** $\mu(A) = 9$ (Total number of prices for Access Bank)

Probability measure: $P(A) = (\text{Number of elements in } A) / (\text{Total number of elements})$

$P(A_{-1}) = 5/9$ (Probability of prices below 100 for Access Bank)

Example 4: **Fidelity Bank:** $\mu(F) = 9$ (Total number of prices for *Fidelity* Bank)

Probability measure: $P(F) = (\text{Number of elements in } F) / (\text{Total number of elements})$

$P(\text{F}_{-2}) = 5/9$ (Probability of prices *above* 100 for *Fidelity* Bank)

We can make the following interpretations for investment decisions: The measurable sets A_{-1} and F_{-1} indicates that banks have a significant number of put option prices below a certain threshold (100 for Access Bank and 80 for Fidelity Bank). This suggests a potential support level for the underlying assets.The probability measures $P(A_{-1})$ and $P(F_{-2})$ provides insights into the likelihood ranges. Investors can use these probabilities to inform their investment decisions, such as determining the likelihood of a put option expiring in- the –money. By analyzing the measurable spaces and measures, investors can better understand the risk profile of their investments. For example, the counting measure $\mu(A)$ and $\mu(F)$ can help investors assess the total number of put option prices and potential risks associated with each bank. These measurable spaces and measures enable quantitative comparisons between the put option prices of Access Bank and Fidelity Bank. Investors can use these comparisons to identify potential arbitrage opportunities or make informed decisions about which banks put options to invest in. Following the interpretations above, investors can buy put options with strike prices near the support levels identified in A_{-1} and F_{-1} . Investors can also sell put options with strike price above the threshold identified in A_{-2} and F_{-2}. Investors can use the measurable spaces and measure to identify potential spread trading opportunity between Access Bank and Fidelity Bank put options. Investors can use the probability measures and counting measures to adjust their investment portfolios and management risk.

Topological Analysis of put option prices: Access Bank and Fidelity Bank.

To apply topological spaces to the put option prices of Access Bank and Fidelity Bank, will examine the properties of the price spaces. Let's denote the put option price spaces as:

Access Bank: $A = \{402, 71, 118, 70, 90, 84, 119, 83, 370\}$

Fidelity Bank: $F = \{407, 52, 139, 51, 113, 72, 131, 71, 376\}$

Topological properties

Connectedness: Both A and F are connected spaces, as the prices are related to the same underlying asset (put options).

Compactness: Both A and F finite sets, making them compact spaces.

Hausdorff property: Both A and F are metric spaces, but they may not be Hausdorff spaces, as distinct prices may not have disjoint neighborhoods.

Topological comparison

To compare the topological properties of A and F ,we can examine their:

Price ranges: Access Bank: $\{70, 402\}$,Fidelity Bank: $F = \{51, 407\}$

Price distributions: Both A and F have varying price distributions, but F seems to have a slightly wider range.

In general, the connectedness and compactness of the price spaces can help investors analyze price movements. Also, recognizing the topological properties of the price spaces can inform risk management strategies.

4.1 CONCLUSION

The topological analysis of put option prices for Access Bank and Fidelity Bank reveals valuable insights into the underlying price dynamics and potential support levels. The measurable spaces and measures define in this study enable quantitative comparisons between the two banks put option prices, facilitating informed investment decisions. Also, our theorems demonstrate that the put option price function is a measurable , integrable, continuous, and homeomorphic function. These findings contribute to the understanding of the behavior of put option prices, providing new insights into the risk management and investment strategies of Access Bank and Fidelity Bank. The key contribution to knowledge is the establishment of a measurable and topological framework for analyzing the put option prices of Access Bank and Fidelity Bank, providing new insights into the behavior of put option prices and informing the development of more accurate and robust option pricing models. This study provides a novel perspective on the comparative analysis of put option prices, and has the potential to inform the development of more effective investment strategies and risk management techniques.

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