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Stochastic Modelling of Heat Transfer in Cylindrical Channels

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ABSTRACT
Non-isothermal flow with stochastic variables remains an aspect of fluid dynamics that sparks great interest in mathematical modeling. But the inability for traditional deterministic models to fully account for the uncertainties especially random fluctuations which affect the non-isothermal conditions under which these fluid systems operate, become a barrier to the model's real-world accuracy This research presents a comprehensive analytical investigation of non-isothermal, incompressible fluid flow through a horizontal cylindrical channel, incorporating stochastic variability in the thermal and solute concentration fields. The study models the unsteady behavior of velocity, temperature, and mass concentration profile by formulating and analyzing coupled partial differential equations (PDEs) governed by momentum equation, energy, and mass transfer respectively. While the momentum equation is treated deterministically, stochastic variables modeled as random perturbations are introduced into the energy and mass concentration equations to account for uncertainties in the boundary conditions and material properties. Using appropriate similarity transformations, the governing equations are non-dimensionalized, yielding coupled ordinary differential equations (ODEs). These are solved using the Frobenius method for homogeneous components and method of undetermined coefficients for non homogeneous components, enabling the derivation of approximate analytical solutions. Key physical parameters including the Grashof number, injection parameter, pressure gradient parameter, andboundary condition parameter are varied to investigate their influence on flow characteristics. The results reveal that stochastic influences significantly modify temperature and mass concentration profiles, especially near boundary layers, while velocity remains largely deterministic. Comparison between deterministic and stochastic approach captures a wider range of fluid behavior, enhancing prediction accuracy under uncertain conditions. This work provides a novel semi-analytical fluid transport in cylindrical systems under stochastic influences.

Keywords: Stochastic fluid, Frobenius, Cylindrical channel, Mass and Differential equations**.**

INTRODUCTION

The mathematical modeling of channel flows has called for great concern to all scholars; as it is expected that this time domain approach will give answers to many problems under different conditions. The modeling of flow in a cylindrical channel is of substantial significance owing to numerous applications in the fast growing field of science and technology. Such applications include the human blood vessels which consist of different networks of arteries. Others include oil and water flow through reservoirs. Channel flow related studies, like, blood flow prediction in human cardiovascular system may help understand some health issues. Some cardiovascular diseases such as ischmia ,atherosclerosis, and angina pectoris are linked to some form of abnormal blood flow in arteries ,[1]. More so, it is imperative to study channel flow problems, especially in cardiovascular system. Physical problem of this nature needs analytic approach, which can give exact solutions for proper mathematical predictions. Nevertheless, stochastic fluid flow in a cylindrical channel can be important in various applications, such as turbulence modeling, environmental engineering, and chemical engineering. In terms of studying turbulence, stochastic fluid flow can help researchers understand the complex and chaotic nature of turbulence in cylindrical channels. Also stochastic fluid flow can be relevance to understanding pollution in cylindrical channels because it can provide insight into the transport and mixing of pollutants in the flow for example, if pollutants are introduced into a cylindrical channel, the stochastic nature of the flow can affect the way they are dispersed and transported. Aforementioned studies have consequently examined such problems but have not fully provided consistent analytical solution with stochastic variables to determine some levels of changes to predict the overall behavior of the system despite the inherent randomness and uncertainty. In particular, some studies, for instance [2] have only focused fluid flow without heat or mass transfer, while some others like [3]who examined the transport of heat induced magnetic field in cylindrical channel etc.

However, the study of non-isothermal flows in cylindrical channels cannot be over emphasized due to its numerous uses in this dynamic area of fluid dynamics : For instance, [4] considered the problem of transport in a Newtonian fluid within a cylindrical domain of non-isothermal flow. In the work of [5] proposed a method to handle both the velocity field and the dimensionless velocity profile of Newtonian and non-Newtonian channel flows in a circular tube.[6] Also [7] carried out a mathematical modeling on composition of non-isothermal CSTR for control. Fuzzy model was applied and results showed little integral absolute error. In 2019, [8] examined four-model approach for blood flow. They applied analytical method and obtained results of variable core viscosity and its field variables in handling the issue of blood flow.[9] studied the effects of viscosity and magnetic field on a non-isothermal cylindrical channel flow.

Nevertheless, the aim of this paper is to examine approximate solutions of a non-isothermal flow in a cylindrical channel with stochastic variables. It is obvious that there must be some environmental effects that may disturb the flow channel at one point to the other; which if not captured cannot realistically simulate a very reliable flow results. This motivated the authors of this paper to incorporate stochastic variables in the model in order to enhance better results for decision making. It is paramount that [4] has considered solution of non-isothermal flow in a cylindrical channel which fluid temperature were coupled to fluid velocity. The advantage of this current paper over[4] is that fluid mass concentration is been coupled to fluid velocity with stochastic variables to capture flow disturbances. Our novel contribution compliments previous efforts in this dynamic area of stochastic fluid dynamics.

This paper is settled as follows: Section 2.1 presents mathematical framework, Section 3,1 presents results and discussion ,the paper is concluded in Section 4.1.

Mathematical Framework

Here, we state some few definitions capturing this area of study

Definition 1.1: Stochastic Processes :A relation of random variables $\{X_t,(r),t \in T$ is a stochastic process $X(t)$. So the variable have t to represents time and $X(t)$ as the state of the process at time t. It also a statistical phenomenon which involves time within the framework of probabilistic principles. In mathematics, a stochastic process entails random variables arranged over time, defined at various time intervals, and can manifest as discrete or continuous.

Definition 1.2: Random Walk: Stochastic process can be stated in different methods such as point $x = a$ to another point $x = b$. Hence a random walk is called a stochastic sequence $\{S_n\}$ with $S_0 = 0$ what is defined as

$$S_n = \sum_{k-1}^{n} X_k \tag{1.1}$$

where X_k represents independent identically distributed random variables.

Definition 1.3: Stochastic Differential Equation (SDE): The dynamics governing SDE is given as follows:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW_t \tag{1.2}$$

Where, μ represents drift and σ is the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $(\Omega, \xi, \wp)$, ξ is a $\sigma -$algebra generated by $W_t, t \geq 0$.

The details of the above concepts can be seen in the works of[10-11] etc.

Problem Formulation

Here, considering an incompressible flow of Newtonian fluid in a cylindrical channel which flows in the horizontal direction z' .The flow is assumed to be dominated along the channel axis, and is taken to be axi-symmetric.The flow is assumed to be non-isothermal with stochastic terms and mass transfer is considered within the flow; No-slip boundary condition is also considered mass concentration and mass concentration with stochastic term have non-zero positive constants on the boundary.Moreover,we assume that the flow is steady and axial symmetric and the stochastic nature of the flow can affect the way they are dispersed and transported. Therefore, Let r be the distance from the channel centre and $\vec{u}, T', C'$ are the fluid velocity mass concentration and mass concentration with stochastic term as shown in Figure 2.1.Let the fluid velocity be $\vec{u} = (0, 0, u')$

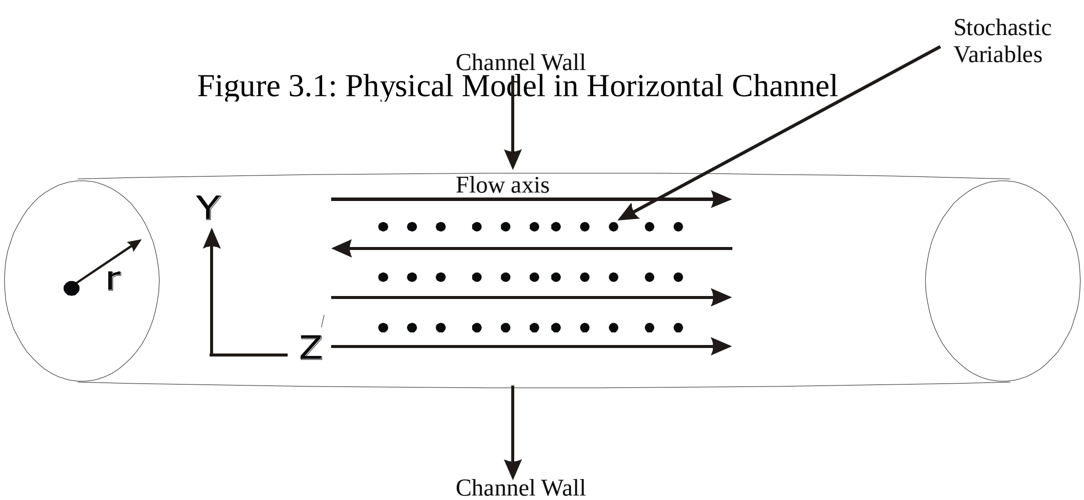

Figure 3.1 : Physical Model in Horizontal Channel

Since the flow is axisymmetric let (r', θ', z') be the cylindrical channel coordinates, where r is the radius of the channel, the directions of flow lies along horizontal axis z' and flow is maintained at non-constant temperature (non-isothermal) see Figure 2, 1.

Based on the assumption of Boussinessq approximation of fluid model and taken consideration of induced magnetic field and steady flow; the equations governing the flow are:

$$\frac{\partial u'}{\partial z'} = 0 \tag{1.3}$$

$$-\frac{\partial p'}{\partial z'} + \mu\left(\frac{\partial^2 u'}{\partial r'^2} + \frac{1}{r'}\frac{\partial u'}{\partial r'}\right) + \rho g \beta_T (T' - T_\infty) - \alpha B'_0 u = 0 \tag{1.4}$$

$$\left(\frac{\partial^2 C'}{\partial r'^2} + \frac{1}{r'}\frac{\partial C'}{\partial r'}\right) + \beta^2 \underline{\alpha}^2 (C' - C_\infty) = 0 \tag{1.5}$$

Subject to the following boundary conditions

$$\frac{\partial u'}{\partial r'} = \frac{\partial T'}{\partial r'} = \frac{\partial C'}{\partial r'} = 0 \tag{1.6}$$

$$u' = 0\ ,\ T' = T_w,\ C' = C_w \text{ on } r' = a \tag{1.7}$$

Where u' w' is the velocity components of fluid, ρ is the fluid density, B_0 is the magnetic field, k is the thermal conductivity, T' is the wall temperature, k is the thermal conductivity, g is the acceleration due to gravity and μ is the viscosity of the fluid, σ is the electrical conductivity, C_p is the heat capacity, q_r is constant magnetic field and β_T is the coefficient volumetric expansion.

Non-dimensionalization

From equations (1.3)-(1.5) and boundary conditions(1.6) and (1.7) respectively the following non-dimensional variables were used:

$$\left.\begin{array}{c} r = \frac{r'}{a} \ , z = \frac{z'}{a}, \theta = \frac{T' - T_\infty}{T_W - T_\infty}, u = \frac{u' a}{v}, \rho = \frac{\rho' a^2}{\mu v}, \\ Pr = \frac{\rho c p a^2}{k} = 1, \quad Gr = \frac{a^3}{\mu v} \rho g \beta_T (T_w - T_\infty), = \underline{\lambda}^2 a^2, \alpha^2 = \underline{\alpha}^2 a^2, \\ M^2 = \frac{\sigma {B_0}^2 a^2}{\mu}, \quad \emptyset = \frac{C' - C_0}{C_W - C_0} \quad \mathrm{P} = \frac{a^3}{\mu v} \frac{\partial p'}{\partial z'} \end{array}\right\} \tag{1.8}$$

where, Gr is the Grashof number.

Substituting (1.8) in (1.3)-(1.7) gives

$$-\frac{\partial p}{\partial z} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \beta_1^2 M^2 u + Gr\theta = 0 = 0 \tag{1.9}$$

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \alpha^2 \phi = 0 \tag{1.10}$$

$$\frac{\partial^2 \phi_0}{\partial r^2} + \frac{1}{r}\frac{\partial \phi_0}{\partial r} + \beta_3^2 \alpha^2 \phi_0 = 0 \tag{1.11}$$

Subject to

$$\frac{\partial u}{\partial r} = \frac{\partial \phi}{\partial r} = \frac{\partial \phi_0}{\partial r} = 0 \tag{1.12}$$

$$u = 0, \phi = \phi_a \ \phi_0 = \phi_a \ 0n \ r = 1 \tag{1.13}$$

Equation(1.3) implies that $u \neq u(z)$,hence u is a function of r alone and we arrived at the following system of ordinary differential equations:

$$\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} - M^2 u = -p - Gr\phi \tag{1.17}$$

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r}\frac{d\phi}{dr} + \alpha^2 \phi = 0 \tag{1.18}$$

$$\frac{d^2\phi_0}{dr^2}+\frac{1}{r}\frac{d\phi_0}{dr}+\beta^2\alpha^2\phi_0=0 \qquad (1.19)$$

Subject to the following boundary conditions

$$\left.\begin{array}{l}\frac{du}{dr}=0,\frac{d\phi}{dr}=0\ ,\frac{d\phi_0}{dr}at\ r=0\\ u=0,\ ,\ \phi=\phi_w,\phi_0=\phi_w at\ r=1\end{array}\right\}$$

(1.20

where p is constant term M represents magnetic field parameter, α represents injection term λ represents cooling term and β. represents stochastic term to determine levels of fluid changes. In the next section, we derive, in detail, an analytical solution of the proposed model.

Method of Solution

The model (1.17)-(1.20) consist of a system of variable coefficient differential equations whose solutions are not trivial. We adopts the method of Frobenius in solving for $u(r,t)$ $\phi(r,t)$ and $\phi_0(r,t)$, To tackle this problem we note that

$u,\quad \phi,\phi_0<\infty$ for all $r\in[0,1)$

Solution of Velocity Model

Also we assume a solution of the form:

$$u=\sum_{m=0}^{\infty}b_m r^{m+k}=r^k\sum_{m=0}^{\infty}b_m r^m \qquad (1.21)$$

$$u'=\sum_{m=0}^{\infty}b_m(m+k)r^{m+k-1}=r^{k-1}\sum_{m=0}^{\infty}b_m(m+k)r^m \qquad (1.22)$$

$$u''=\sum_{n=0}^{\infty}am(m+k)r^{m+k-2}=r^{k-2}\sum_{n=0}^{\infty}am(m+k)(m+k-1)r^m$$

$$\Rightarrow ru''=r^{k-1}\sum_{n=0}^{\infty}am(m+k)(n+k-1)r^m \qquad (1.23)$$

Putting (1.21)-(1.23) into (1.17) gives

$$r^{k-1}\sum_{m=0}^{\infty}b_m(m+k)(m+k-1)r^m+r^{k-1}\sum_{m=0}^{\infty}b_m(m+k)r^m-\sum_{m=0}^{\infty}b_m r^{m+k+1}=0 \qquad (1.24)$$

$$r^{k-1}\sum_{m=0}^{\infty}b_m(m+k)^2 r^m-\sum_{m=0}^{\infty}M^2 b_m r^{m+k+1}=0 \qquad (1.25)$$

$$r^{k-1}b_0k^2+\sum_{m=0}^{\infty}b_m\left(m+k\right)^2r^{m+k-1}-\sum_{m=0}^{\infty}b_mM^2r^{m+k+1}=0$$

Setting $q=m-1\Rightarrow m=q+1$

$$r^{k-1}b_0k^2+\sum_{k=0}^{\infty}b_{q+1}\left(q+k+1\right)^2r^{k+1}-\sum_{m=0}^{\infty}b_mM^2r^{m+k+1}=0$$

Equating coefficients of indicial equation and therefore performing some algebraic operations gives

$$\begin{aligned}u=b_0r^k\Bigg\{&1+\frac{M^2r^2}{\left(k+2\right)^2}+\frac{M^4r^4}{\left(k+2\right)^2\left(k+4\right)^2}+\frac{M^6r^6}{\left(k+2\right)^2\left(k+4\right)^2\left(k+6\right)^2}\\&+\frac{M^8r^8}{\left(k+2\right)^2\left(k+4\right)^2\left(k+6\right)^2\left(k+8\right)^2}+\ldots\Bigg\}\end{aligned}\qquad(1.26)$$

When $k=0$ (1.26) becomes

$$u=u_1=A_1\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right\}\qquad(1.27)$$

A linear combination (1.26) and(1.27) gives the complete solution

$$\begin{aligned}u=&A_1\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}\right\}\\&+A_2\Bigg\{\ln r\left(1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)\\&-\frac{M^2r^2}{2^2}-\frac{M^4r^4}{2^3\times4^2}-\frac{M^6r^6}{4^3\times6^3}+\frac{M^8r^8}{2^5\times6^3\times8^3}-\ldots\Bigg\}\end{aligned}\qquad(1.28)$$

Applying the boundary conditions (1.20) and setting $A_2=0$ in (1.28) gives

$$A_1=\frac{\theta_a}{y_1(\mathrm{a})}\qquad(1.29)$$

where $y_{1(\mathrm{a})}=\left(1+\frac{M^2a^2}{2^2}+\frac{M^4a^4}{2^2\times4^2}+\frac{M^6a^6}{2^2\times4^2\times6^2}+\frac{M^8a^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right),$

$$u=\frac{\theta_a}{y_1(1)}\left\{1+\frac{M^2r^2}{2^2}+\frac{M^4r^4}{2^2\times4^2}+\frac{M^6r^6}{2^2\times4^2\times6^2}+\frac{M^8r^8}{2^2\times4^2\times6^2\times8^2}+\cdots\right\}\qquad(1.30)$$

Solution of Mass Concentration Model

From (1.18)
we assume a solution of the form :

$$\phi = \sum_{m=0}^{\infty} b_m r^{m+k} = r^k \sum_{m=0}^{\infty} b_m r^m \quad (1.31)$$

$$\phi' = \sum_{m=0}^{\infty} b_m \left(m+k\right) r^{m+k-1} = r^{k-1} \sum_{m=0}^{\infty} b_m \left(m+k\right) r^m \quad (1.32)$$

$$\phi'' = \sum_{n=0}^{\infty} am\left(m+k\right) r^{m+k-2} = r^{k-2} \sum_{n=0}^{\infty} am\left(m+k\right)\left(m+k-1\right) r^m$$

$$\Rightarrow r\phi'' = r^{k-1} \sum_{n=0}^{\infty} am\left(m+k\right)\left(n+k-1\right) r^m \quad (1.33)$$

Putting (1.31)-(1.33) into (1.18) gives

$$r^{k-1} \sum_{m=0}^{\infty} b_m \left(m+k\right)\left(m+k-1\right) r^m + r^{k-1} \sum_{m=0}^{\infty} b_m \left(m+k\right) r^m + \sum_{m=0}^{\infty} \alpha^2 b_m r^{m+k+1} = 0 \quad (1.34)$$

$$r^{k-1} \sum_{m=0}^{\infty} b_m \left(m+k\right)^2 r^m + \sum_{m=0}^{\infty} \alpha^2 b_m r^{m+k+1} = 0 \quad (1.35)$$

$$r^{k-1} b_0 k^2 + \sum_{m=0}^{\infty} b_m \left(m+k\right)^2 r^{m+k-1} + \sum_{m=0}^{\infty} b_m \alpha^2 r^{m+k+1} = 0$$

Setting $q = m-1 \Rightarrow m = q+1$

$$r^{k-1} b_0 k^2 + \sum_{k=0}^{\infty} b_{q+1} \left(q+k+1\right)^2 r^{k+1} + \sum_{m=0}^{\infty} b_m \alpha^2 r^{m+k+1} = 0$$

Equating coefficients of indicial equation, and therefore performing some algebraic operations gives

$$\phi = u_1 = C_1 \left\{ 1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \ldots \right\} \quad (1.36)$$

Another is given by $v_1 = \dfrac{d\phi}{dk}$

$$\frac{d\phi}{dk} = b_0 r^k lnr \left\{1 - \frac{\alpha^2 r^2}{(k+2)^2} + \frac{\alpha^4 r^4}{(k+2)^2 (k+4)^2} - \frac{\alpha^6 r^6}{(k+2)^2 (k+4)^2 (k+6)^2} + \frac{\alpha^8 r^8}{(k+2)^2 (k+4)^2 (k+6)^2 (k+8)^2} + \dots \right\}$$

$$+ b_0 r^k \frac{d}{dk} \left\{1 - \frac{\alpha^2 r^2}{(k+2)^2} + \frac{\alpha^4 r^4}{(k+2)^2 (k+4)^2} - \frac{\alpha^6 r^6}{(k+2)^2 (k+4)^2 (k+6)^2} + \frac{\alpha^8 r^8}{(k+2)^2 (k+4)^2 (k+6)^2 (k+8)^2} + \dots \right\}$$

$$\frac{d\phi}{dk} = b_0 r^k lnr \left\{1 - \frac{\alpha^2 r^2}{(k+2)^2} + \frac{\alpha^4 r^4}{(k+2)^2 (k+4)^2} - \frac{\alpha^6 r^6}{(k+2)^2 (k+4)^2 (k+6)^2} + \frac{\alpha^8 r^8}{(k+2)^2 (k+4)^2 (k+6)^2 (k+8)^2} + \dots \right\} + b_0 r^k \left\{0 + \frac{2\alpha^2 r^2}{(k+2)^3} - \frac{4\alpha^4 r^4 (k+3)}{(k+2)^3 (k+4)^3} + \frac{8\alpha^6 r^6}{(k+2)^3 (k+4)^3 (k+6)^3} + \frac{16\alpha^8 r^8}{(k+2)^3 (k+4)^3 (k+6)^3 (k+8)^3} - \dots \right\}$$

When

$$k = 0$$

$$\phi = v_2 = C_2 \left\{ \ln r \left(1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dots \right) + b_0 r^k \left\{ + \frac{\alpha^2 r^2}{2^2} - \frac{\alpha^4 r^4}{2^3 \times 4^2} + \frac{\alpha^6 r^6}{4^3 \times 6^3} + \frac{\alpha^8 r^8}{2^5 \times 6^3 \times 8^3} - \dots \right\} \right. \tag{1.37}$$

A linear combination (1.36) and (1.37) gives the complete solution

$$\phi = C_1 \left\{1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dots \right\} + C_2 \left\{ \ln r \left(1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \dots \right) + \frac{\alpha^2 r^2}{2^2} - \frac{\alpha^4 r^4}{2^3 \times 4^2} + \frac{\alpha^6 r^6}{4^3 \times 6^3} + \frac{\alpha^8 r^8}{2^3 \times 2^2 \times 6^3 \times 8^3} - \dots \right\} \tag{1.38}$$

Applying the boundary conditions (1.20) then setting $C_2 = 0$ in (1.38) gives

$$C_1 = \frac{\phi_a}{y^*{}_1(\mathrm{a})} \qquad (1.39)$$

where $\quad y^*{}_{1(\mathrm{a})} = \left(1 - \frac{\alpha^2 a^2}{2^2} + \frac{\alpha^4 a^4}{2^2 \times 4^2} - \frac{\alpha^6 a^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 a^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \ldots\right),$

$$\phi = \frac{\phi_a}{y^*{}_1(\mathrm{a})}\left\{1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} + \cdots\right\} \qquad (1.40)$$

Non-homogenous part of Veloccity Model

To get the particular solution of (1.17) which is the non-homogenous part of the problem, we solve using the method of undetermined coefficients.

$$y(r) = A\left\{1 - \frac{\alpha^2 r^2}{2^2} + \frac{\alpha^4 r^4}{2^2 \times 4^2} - \frac{\alpha^6 r^6}{2^2 \times 4^2 \times 6^2} + \frac{\alpha^8 r^8}{2^2 \times 4^2 \times 6^2 \times 8^2} - \cdots\right\}, A \in \mathbb{R}$$

The complementary function for (1.17) is

$$u_c(r) = C_c\left[1 + \frac{(Mr)^2}{2^2} + \frac{(Mr)^4}{(2\times 4)^2} + \frac{(Mr)^6}{(2\times 4\times 6)^2} + \frac{(Mr)^8}{(2\times 4\times 6\times 8)^2} + \ldots\right] \qquad (1.41)$$

Consider the particular solution

$$u_p(r) = A_0 + A_1 r^2 + A_2 r^4 + A_3 r^6 + A_4 r^8 \qquad (1.42)$$

$$u_p'(r) = 2A_1 r + 4A_2 r^3 + 6A_3 r^5 + 8A_4 r^7 \qquad (1.43)$$

$$u_p''(r) = 2A_1 + 12A_2 r^2 + 30A_3 r^4 + 56A_4 r^6 \qquad (1.44)$$

Putting (1.42)-(1.44) into (1.17) gives

$$\Rightarrow 4A_1 + 16A_2 r^2 + 36A_3 r^4 + 64A_4 r^6 - M^2 A_0 - M^2 A_1 r^2 - M^2 A_2 r^4 - M^2 A_3 r^6 - M^2 A_4 r^8 \equiv$$
$$-p - Gr\phi\left(1 - \frac{(\alpha r)^2}{2^2} + \frac{(\alpha r)^4}{(2\times 4)^2} - \frac{(\alpha r)^6}{(2\times 4\times 6)^2} + \frac{(\alpha r)^8}{(2\times 4\times 6)^2}\right) \qquad (1.45)$$

$$RHS = \left(4A_1 - M^2 A_0\right) + \left(16A_2 - M^2 A_1\right) r^2 + \left(36A_3 - M^2 A_2\right) r^4 + \left(64A_4 - M^2 A_3\right) r^6 + (-M^2 A_4)\,\mathrm{r}^8 \qquad (1.46)$$

Combining (1.45) and (1.46) and performing some algebraic operations yields the following results

$$u(r) = c + A_0 + \left(\frac{cM^2}{4} + A_1\right)r^2 + \left(\frac{cM^4}{(2\times 4)^2} + A_2\right)r^4 + \left(\frac{cM^6}{(2\times 4\times 6)^2} + A_3\right)r^6 + \left(\frac{cM^8}{(2\times 4\times 6\times 8)^2} + A_4\right)r^8 \quad (1.47)$$

Solution of Concentration Model with Stochastic parameter

Similarly from (1.19) we assume a solution of the form :

$$\phi_0 = \sum_{m=0}^{\infty} b_m r^{m+k} = r^k \sum_{m=0}^{\infty} b_m r^m \quad (1.48)$$

$$\phi_0' = \sum_{m=0}^{\infty} b_m (m+k) r^{m+k-1} = r^{k-1}\sum_{m=0}^{\infty} b_m (m+k) r^m \quad (1.49)$$

$$\phi_0'' = \sum_{n=0}^{\infty} am(m+k) r^{m+k-2} = r^{k-2}\sum_{n=0}^{\infty} am(m+k)(m+k-1) r^m$$

$$\Rightarrow r\phi_0'' = r^{k-1}\sum_{n=0}^{\infty} am(m+k)(n+k-1) r^m \quad (1.50)$$

Putting (1.48)-(1.50) into (1.19) gives

$$r^{k-1}\sum_{m=0}^{\infty} b_m (m+k)(m+k-1) r^m + r^{k-1}\sum_{m=0}^{\infty} b_m (m+k) r^m + \sum_{m=0}^{\infty} \beta_3^2 \alpha^2 b_m r^{m+k+1} = 0 \quad (1.51)$$

$$r^{k-1}\sum_{m=0}^{\infty} b_m (m+k)^2 r^m + \sum_{m=0}^{\infty} \beta_3^2 \alpha^2 b_m r^{m+k+1} = 0 \quad (1.52)$$

$$r^{k-1} b_0 k^2 + \sum_{m=0}^{\infty} b_m (m+k)^2 r^{m+k-1} + \sum_{m=0}^{\infty} b_m \beta_3^2 \alpha^2 r^{m+k+1} = 0$$

Setting $q = m-1 \Rightarrow m = q+1$

$$r^{k-1} b_0 k^2 + \sum_{k=0}^{\infty} b_{q+1} (q+k+1)^2 r^{k+1} + \sum_{m=0}^{\infty} b_m \beta_3^2 \alpha^2 r^{m+k+1} = 0$$

Equating coefficients of indicial equation, and therefore performing some algebraic operations gives

$$\phi_0 = u_1 = C_1\left\{1-\frac{\beta_3^2\alpha^2r^2}{2^2}+\frac{\beta_3^4\alpha^4r^4}{2^2\times4^2}-\frac{\beta_3^6\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\dots\right\} \quad (1.53)$$

Another is given by $v_1 = \frac{d\phi_0}{dk}$

$$\frac{d\phi_0}{dk} = b_0r^k lnr\left\{1-\frac{\beta_3^2\alpha^2r^2}{(k+2)^2}+\frac{\beta_3^4\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\beta_3^6\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}+\frac{\beta_3^8\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\dots\right\}$$
$$+b_0r^k\frac{d}{dk}\left\{1-\frac{\beta_3^2\alpha^2r^2}{(k+2)^2}+\frac{\beta_3^4\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\beta_3^6\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}+\frac{\beta_3^8\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\dots\right\}$$

$$\frac{d\phi_0}{dk} = b_0r^k lnr\left\{1-\frac{\beta_3^2\alpha^2r^2}{(k+2)^2}+\frac{\beta_3^4\alpha^4r^4}{(k+2)^2(k+4)^2}-\frac{\beta_3^6\alpha^6r^6}{(k+2)^2(k+4)^2(k+6)^2}+\frac{\beta_3^8\alpha^8r^8}{(k+2)^2(k+4)^2(k+6)^2(k+8)^2}+\dots\right\}+b_0r^k\left\{0+\frac{2\beta_3^2\alpha^2r^2}{(k+2)^3}-\frac{4\beta_3^4\alpha^4r^4(k+3)}{(k+2)^3(k+4)^3}\right.$$ When

$$\left.+\frac{8\beta_3^6\alpha^6r^6}{(k+2)^3(k+4)^3(k+6)^3}+\frac{16\beta_3^8\alpha^8r^8}{(k+2)^3(k+4)^3(k+6)^3(k+8)^3}-..\right\}$$

$k=0$

$$\phi_0 = v_2 = C_2\left\{\ln r\left(1-\frac{\beta_3^2\alpha^2r^2}{2^2}+\frac{\beta_3^4\alpha^4r^4}{2^2\times4^2}-\frac{\beta_3^6\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\dots\right)\right. \quad (1.54)$$
$$\left.+b_0r^k\left\{+\frac{\beta_3^2\alpha^2r^2}{2^2}-\frac{\beta_3^4\alpha^4r^4}{2^3\times4^2}+\frac{\beta_3^6\alpha^6r^6}{4^3\times6^3}+\frac{\beta_3^8\alpha^8r^8}{2^5\times6^3\times8^3}-\dots\right\}\right.$$

A linear combination (1.53) and (1.54) gives the complete solution

$$\phi_0 = C_1\left\{1-\frac{\beta_3^2\alpha^2r^2}{2^2}+\frac{\beta_3^4\alpha^4r^4}{2^2\times4^2}-\frac{\beta_3^5\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right\}$$
$$+C_2\left\{\ln r\left(1-\frac{\beta_3^2\alpha^2r^2}{2^2}+\frac{\beta_3^4\alpha^4r^4}{2^2\times4^2}-\frac{\beta_3^6\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)\right. \quad (1.55)$$
$$\left.+\frac{\beta_3^2\alpha^2r^2}{2^2}-\frac{\beta_3^4\alpha^4r^4}{2^3\times4^2}+\frac{\beta_3^6\alpha^6r^6}{4^3\times6^3}+\frac{\beta_3^8\alpha^8r^8}{2^3\times2^2\times6^3\times8^3}-\ldots\right\}$$

Applying the boundary conditions (1.20) then setting $C_2 = 0$ in (1.55) gives

$$C_1 = \frac{\phi_a}{y^*_1(\mathrm{a})} \quad (1.56)$$

where $y^*_{1(\mathrm{a})} = \left(1-\frac{\beta_3^2\alpha^2a^2}{2^2}+\frac{\beta_3^4\alpha^4a^4}{2^2\times4^2}-\frac{\beta_3^6\alpha^6a^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8a^8}{2^2\times4^2\times6^2\times8^2}+\ldots\right)$,

$$\phi_0 = \frac{\phi_a}{y^*_1(\mathrm{a})}\left\{1-\frac{\beta_3^2\alpha^2r^2}{2^2}+\frac{\beta_3^4\alpha^4r^4}{2^2\times4^2}-\frac{\beta_3^6\alpha^6r^6}{2^2\times4^2\times6^2}+\frac{\beta_3^8\alpha^8r^8}{2^2\times4^2\times6^2\times8^2}+\cdots\right\} \quad (1.57)$$

RESULTS AND DISCUSSION

This Section we present the computational results for the problem formulated equations. The solutions are implemented in a python programming

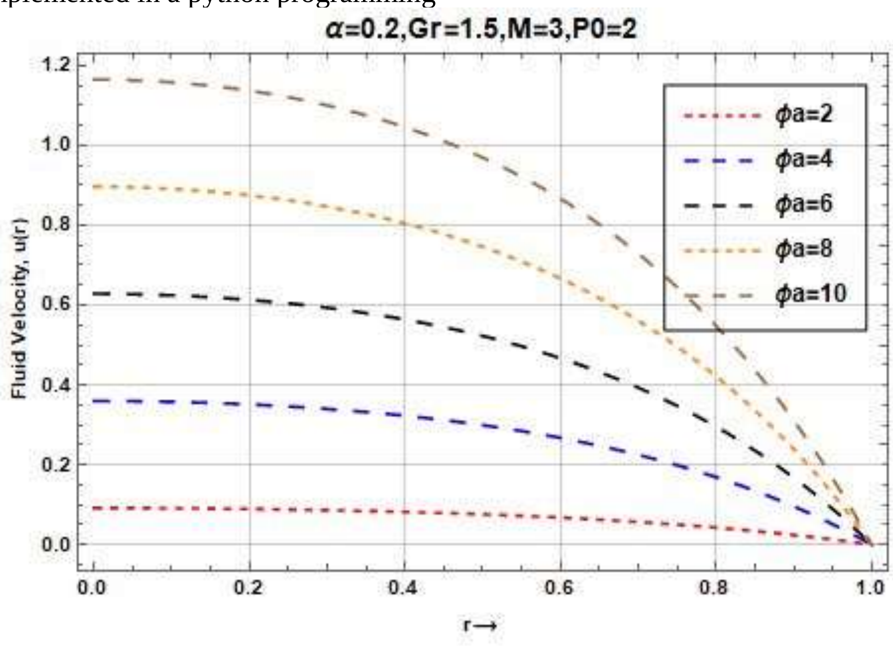

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Figure 1.1: Fluid Velocity profiles against r with variations of boundary condition parameter

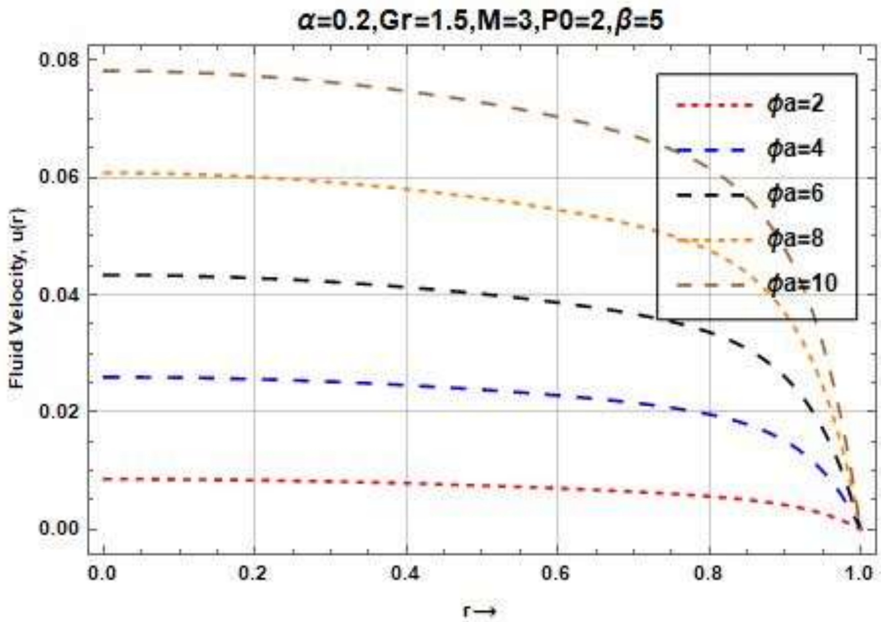

Figure 1.2: Stochastic Fluid Velocity profiles against r with variations of boundary condition parameter

In Figures 1.2 and 1.3 connotes increase in boundary condition parameter increases fluid velocity. Changes in the shape, size or arrangement of boundary indeed affect the velocity of the fluid . Example, if you were to increase the size of a boundary in a flow system, the increased boundary area could lead to a higher momentum exchange between the boundary and bulk flow, resulting in an increase increase in fluid velocity.

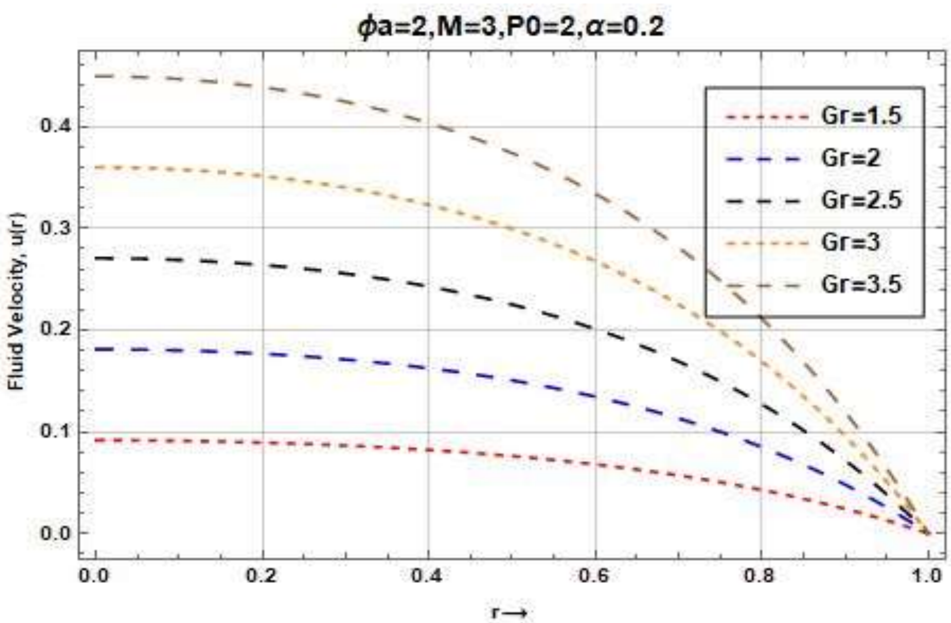

Figure 1.3: Fluid Velocity profiles against r with variations of Grashof number

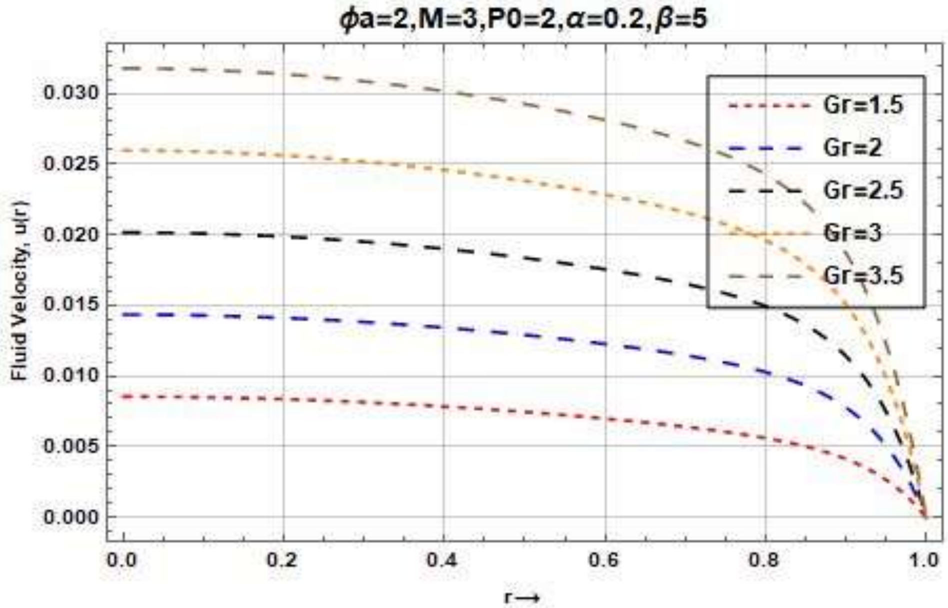

Figure 1.4: Stochastic Fluid Velocity profiles against r **with variations of Grashof number**

Figures 1.3 and 1.4 shows the variation of fluid velocity with Grashofnumber . It is clear that an increase in Grashof number leads to an increase in fluid velocity. This implies that extra heat is added to the system by increasing the Grashof number. Which agrees with the results of [4],[12],[13] etc.

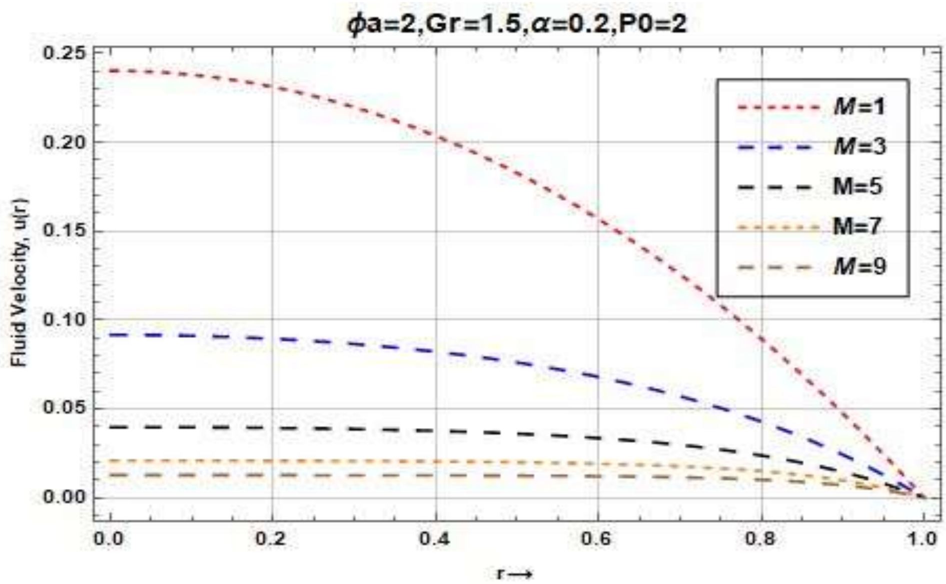

Figure 1.5: Fluid Velocity profiles against r **with variations of Magnetic field parameter**

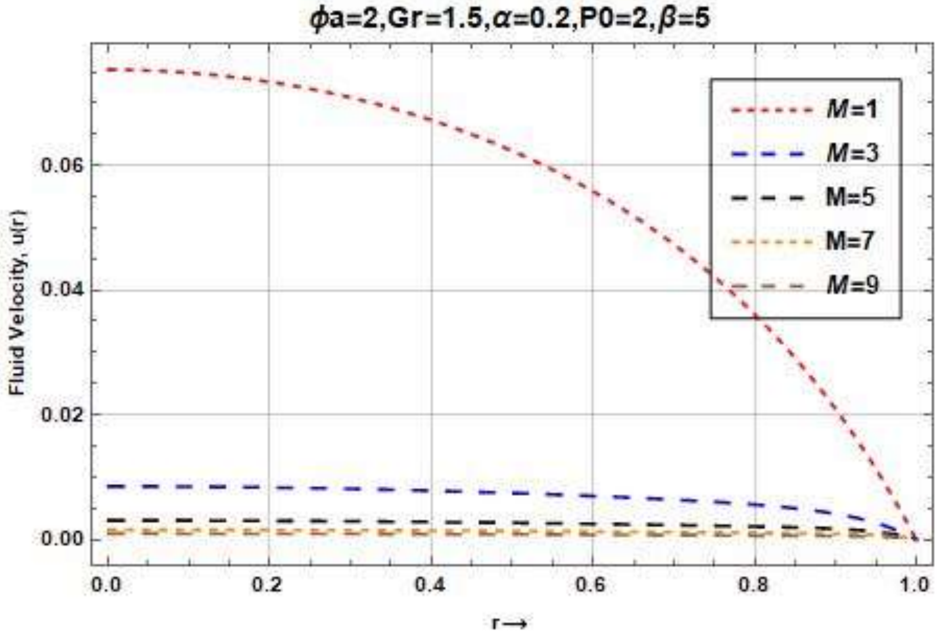

Figure 1.6: Stochastic Fluid Velocity profiles against r with variations of Magnetic field parameter

Figures 1.6 and 1.7 shows the variation of fluid velocity with magnetic field parameter. It can be seen that increase in the magnetic field parameter decreases the fluid velocity. In practical point of view, the occurrence of magnetic field leads the Lorentz force which acts against the flow, therefore decreases in velocity. Which is in accordance with the results of [16], [17] and [18] etc.

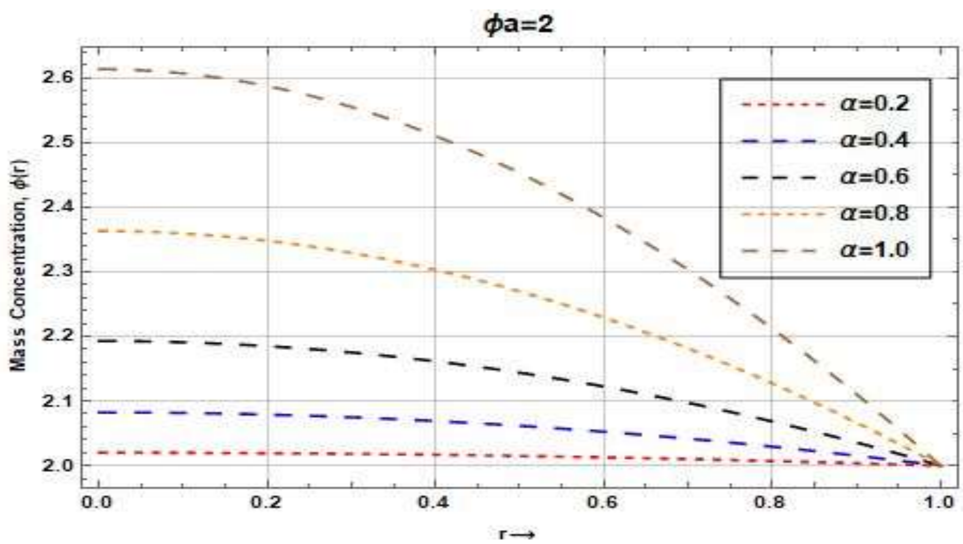

Figure 1.7: Mass Concentration profiles against r with variations of injection parameter

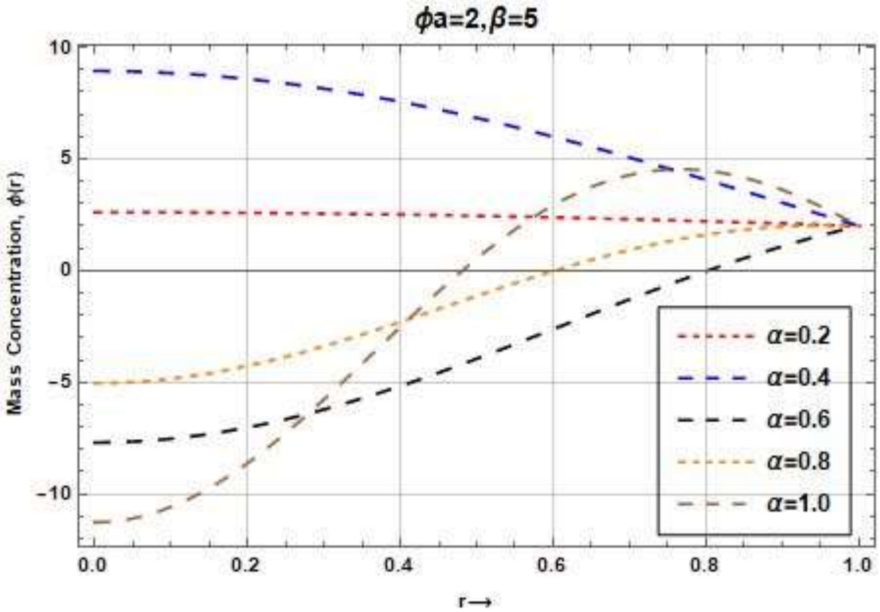

Figure 1.8: Stochastic Mass Concentration profiles against r with variations of injection parameter

It is shown in Figures 1.7 and 1.8 that an increase in the injection parameter increases fluid concentration. This is physically obvious in the sense that an increasing injection parameter would increase the level by which the pollutant is being added into the fluid; this is also similar with increasing boundary condition parameter in Figures 1.9 and 1.10. Therefore, it is realistic. This is in line with result of [5] and [4].

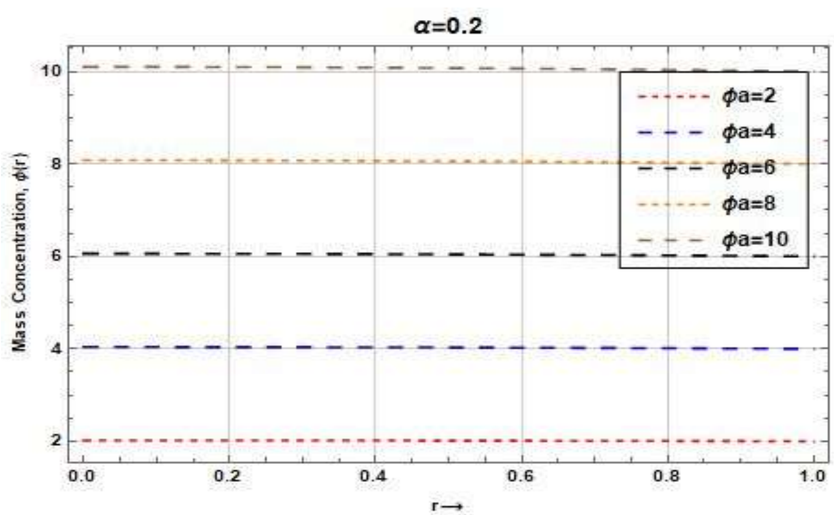

Figure 1.9: Mass Concentration profiles against r with variations of boundary condition parameter

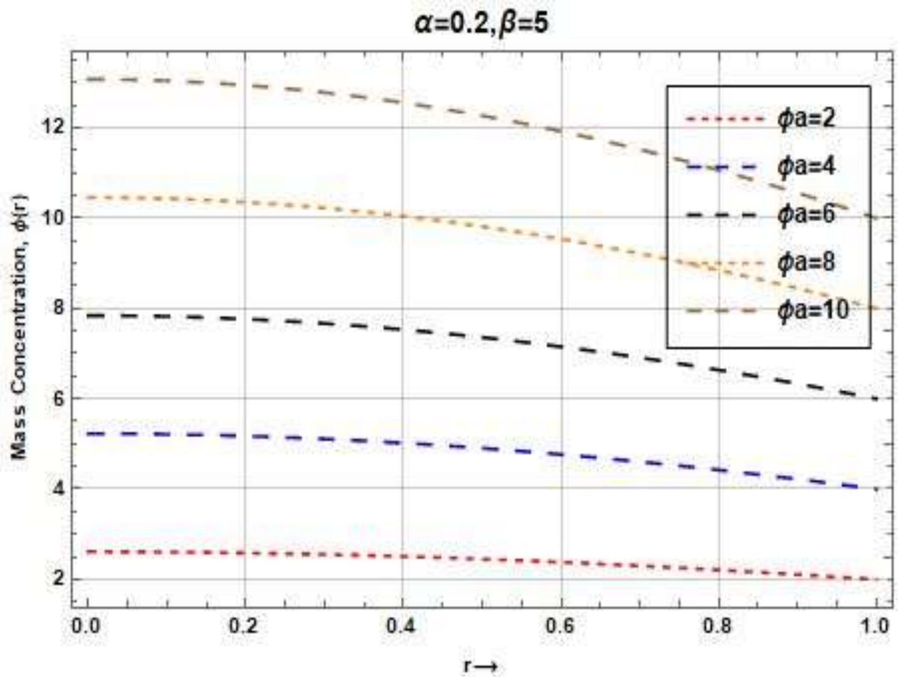

Figure 1.10: Stochastic Mass Concentration profiles against r **with variations of boundary condition parameter**

However, In stochastic fluid velocity, stochastic fluid temperature and stochastic fluid mass concentration profiles such as Figures: 1.2, 1.4, 1.6, 1.8 and 1.10 shows stochastic effects which refers to random or unpredictable variations in a process. In the context of these distributions in cylindrical channels, the stochastic effects could arise from a variety of sources. For example, turbulent fluctuations in the flow, small-scale fluctuations in the flow, small-scale fluctuations in the surface roughness of the channel walls or fluctuations in the boundary conditions such as fluctuations in wall velocity, wall temperature or wall mass concentration could all lead to stochastic variations in fluid. These variations would typically manifest as fluctuations in the local velocity, local temperature and local concentration at a given point in channel, as well as fluctuations in the time-averaged velocity, temperature and concentration across the channels

CONCLUSION

The modeling of non-isothermal flow in a cylindrical channel is of substantial significance owing to numerous presentations in the fast growing field of science and technology. In this paper, we investigated the problem of fluid velocity, mass concentration with stochastic parameters in a cylindrical channel flow which assumed axi-symmetric along the channel axis. A coupled system of three differential equations is proposed for the problem and three analytical solutions which mimic the distributions of the flow variables were well presented graphically in different forms to look like what they represent. More so, the fluid velocity versus stochastic fluid velocity and fluid mass concentration versus stochastic mass concentration were compared accordingly. The unpredicted variations in the stochastic process in the channel flows shows different turbulent fluctuations; which captures flow disturbances in the cylindrical channels. In all, flow variables with or without stochastic parameters are all efficient in this practical scenarios. This is very informative in terms of decision making.

Nomenclatures

u' := Dimensional velocity components

z' := Dimensional axisymetric flow of the channel

r' := Dimensional radius

t' := Dimensional time

T' := Dimensional fluid temperature

T'_w := Dimensional wall temperature

ρ := Fluid density

M := Magnetic field parameter

k := Thermal conductivity

g := Acceleration due to gravity

μ := viscosity of the fluid

σ := Electrical conductivity

C_p := Heat capacity

β := coefficient of volumetric expansion

q'_r := Radiative heat flux

α := Concentration injection parameter

λ := Velocity cooling parameter

λ := Temperature cooling parameter

θ_a := Temperature boundary parameter

φ_a := Temperature boundary parameter

Gr := Grashof number

β := Stochastic parameter

Pr : Pressure gradient

$u_0(r) :=$ Dimensionless velocity

$u(r) :=$ Velocity profile of the fluid

$\theta_0'(r) :=$ Dimensionless temperature

$\theta(r) :=$ Temperature profile

$\theta_0(r) :=$ Stochastic Temperature profile

$\phi_0'(r) :=$ Dimensionless concentration

$\phi(r) :=$ Concentration profile

$\phi_0(r) :=$ Stochastic Concentration profile

$Gr :=$ Grashof number

$T_\infty :=$ Free stream temperature

$\tau_w :=$ Shear stress at the wall

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