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A Comparative Convergence Analysis of Variational Iteration and Adomian Decomposition Methods for Nonlinear Gas Dynamic Equations

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ABSTRACT

Nonlinear gas dynamic equations play a crucial role in modeling compressible fluid flow in various scientific and engineering applications. Due to their nonlinear nature, obtaining exact analytical solutions is often challenging, thereby necessitating the use of efficient approximate methods. This study presents a comparative convergence analysis of two prominent semi-analytical techniques: the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM). Both methods are applied to selected nonlinear gas dynamic equations to evaluate their convergence behavior, computational efficiency, and accuracy. The results demonstrate that while both methods are effective, VIM exhibits faster convergence in certain cases, whereas ADM provides more structured approximations through decomposition. Numerical simulations and error analyses are included to support the findings.

Key words: Nonlinear Gas Dynamic Equations, Variational Iteration Method (VIM), Adomian Decomposition Method (ADM), Convergence Analysis, Approximate Solutions, Error Analysis.

INTRODUCTION

Gas dynamics is the science of flow of air and other gas or the motion of bodies through the air and other gas and its effects on physical systems, fluid mechanics and thermodynamics. The equations of gas dynamics are mathematical expressions based on the natural laws of conservation (mass, momentum and energy). The three major nonlinear wave equations that describe ideal gas dynamics behaviours are shock fronts, rare fractions and contact discontinuities (Aminikhah and Jamaican, 2023). Studies in gas dynamics coincide with the developments of transonic and supersonic flight. Many studies have shown that compressible and high speed flows abound in theories of gas dynamics after world war II (Nikkarn 2012). Nonlinear partial differential equations (PDEs) frequently arise in gas dynamics, especially in modeling compressible flows, shock waves, and turbulence phenomena (Wazwaz, 2009). These equations are often difficult to solve analytically due to their strong nonlinearities. Several researchers have proposed several analytical and numerical solutions for nonlinear gas dynamics equations. Hossein et al (2009) provided analytical solutions for nonlinear gas dynamics equations using homotopy analysis method, while Adesina (2017) used natural homotopy perturbation method to solve gas dynamics equation. Evans and Bulut (2002) and Shehu and Sabuwa (2014) both employed efficient techniques of natural decomposition to solve some gas dynamics equations. Also, Premkiran and Pradhan (2013)

presented and applied Elzaki transform homotopy perturbation method for solving gas dynamics equations. Mohiuddin (2015) presented a modified homotopy perturbation method (MHPM) for dynamics gas equations.

On the use of approximate methods for the solutions of applied differential equations, Rasular and Karaguler (2003) presented some finite difference schemes for solving system equations of gas dynamic, while the Laplace variation iteration method was used to solve the nonlinear gas dynamics equation by Joseph et al (2020). In the same vein, Njoseh (2016) presented a numerical method called the Variation iteration Adomian decomposition method (VIADM) for solving nonlinear partial differential equations (PDEs). The method modifies the traditional formulation of the variation iteration decomposition method (VIDM) such that it converges more rapidly to the analytic solution. Mamadu et al (2025) developed the Mamadu Variational Iteration Method (MVIM) as a powerful numerical toolbox for obtaining analytical and approximate solutions to the time-fractional telegraph equation. The method involves constructing an iterative correction functional that incorporates Caputo fractional derivatives and Mamadu weighting, ensuring rapid convergence and improved accuracy compared to the classical VIM

More recently, approximate methods have been employed to provide solutions to model simulational studies in numerous aspects of mathematical modelling. Mamadu and Apanapudor (2017) employed variation iteration decomposition method for analytic solution of gas dynamic equation. Falafel (2022) applied the Algorithmic analytic numerical solution for nonlinear gas dynamic partial differential equation.

Besides gas equations, other fields of applications have been widely explored by use of approximate numerical methods and simulations. For instance, Mamadu et al (2020) used a numerical approach to analyse an SEIR compartmental disease transmission model, while Onyemarin et al (2023) used a time series model with numerical simulations to analyse infant mortality rates in some regions of Nigeria. Also, Ehiwario et al (2023) coupled a lifetime probability distribution with established survival and hazard rate functions and used approximate methods to estimate recovery and mortality rates arising from the Covid-19 pandemic in Nigeria. Nduka and Igabari (2007) proposed a generating function that could generate both raw and central moments, while Matthew et al (2017) developed a general method for obtaining such moments. In epidemiological studies of populations, Aghanenu et al (2022) investigated the effect of control strategies in epidemic management using a numerically simulated model. Igabari (2017) also used approximate methods to compare models of Nigeria population growth. Other efforts include Osemeke et al (2024), who used numerical illustrations to show the role of variance inflation factor (VIF) in detecting model assumption violations and multi-collinearity, and Onyeoghane et al (2025), who presented a Petrov–Galerkin Finite Element Method, perturbed by the Mamadu–Njoseh Orthogonal Polynomials for the solution of the space time fractional Fitzhugh–Nagumo equations. Both Esokpor and Igabari (2026), and Igabari and Esokpor (2026) established that the integration of topological data analysis with machine learning will aid predictive accuracy and faster convergence of iterative schemes.

Traditional numerical methods such as finite difference and finite element methods have been widely used, but they often suffer from discretization errors, stability constraints and convergence issues (Evans and Raslan, 2005). Consequently, semi-analytical methods like the Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) have gained significant attention due to their efficiency and reduced computational cost.

The Variational Iteration Method, introduced by He (1999), provided the needed iterative correction through Lagrange multipliers. On the other hand, the Adomian Decomposition Method, developed by Adomian (1988), decomposes nonlinear operators into polynomial series, enabling systematic approximation of solutions.

In this paper, we shall be considering a comparative convergence Analysis of Variational Iteration and Adomian Decomposition Method for the solution of nonlinear gas dynamic equations.

Problem Formulation

In this paper, we consider the nonlinear gas dynamics equations in one spatial dimension of the form

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2} \tag{1}$$

With the initial condition $U(x, 0) = \sin(x)$ (2)

Where $U(x, 0)$ and $\sin(x)$ are smooth functions.

Suppose $V = 0$, then (1) is said to be homogeneous. We focus on the comparative convergence analysis of two prominent semi-analytical techniques: the Variational Iteration Method (VIM) and the Adomian Decomposition Method, computational efficiency, and accuracy of VIM and ADM when applied to nonlinear gas dynamic equations.

METHODOLOGY

Consider a general nonlinear gas dynamic equation of the form:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2} ; x \in R, t > 0 \tag{3}$$

Where $U(x, t)$ is the velocity field and V is the viscosity coefficient.

This equation corresponds to the well-known Burgers' equation, which is widely used in gas dynamics.

The Variational Iteration Method (VIM) is one of the popular iterative schemes used for solving both linear and non-linear differential equations. It employs an iterative approach to obtain approximate solutions through successive approximations. The Variational Iteration Method constructs correction functionals using Lagrange multipliers. The general formulation for a nonlinear equation is given as

$$Lu + Nu = g(x, t) \tag{4}$$

Where L is a linear operator and N is a nonlinear operator.

Then the VIM correction functional can be expressed as;

$$U_{n+1}(x, t) = U_n(x, t) + \int_0^t \lambda(\tau) [LU_n + NU_n - g] d\tau \tag{5}$$

Where $\lambda(\tau)$ is the Lagrange multiplier and $U_n(x, t)$ is the n th approximation of the solution.

Applying VIM to the nonlinear gas dynamic equation yields an iterative scheme:

$$U_{n+1} = U_n + \int_0^t \lambda \left(\frac{\partial U_n}{\partial \tau} + U_n \frac{\partial U_n}{\partial x} - v \frac{\partial^2 U_n}{\partial x^2} \right) d\tau \tag{6}$$

The Lagrange multiplier is optimally identified using variational theory. (6) is in its differential form.

The Adomian Decomposition Method (ADM) decomposes the solution and nonlinear terms into infinite series. The solution of Adomian Decomposition Method is expressed as:

$$U(x, t) = \sum_{n=0}^{\infty} U_n(x, t) \tag{7}$$

while the nonlinear term is decomposed using Adomian polynomials is given as

$$U_n = \sum_{n=0}^{\infty} A_n \tag{8}$$

where A_n are Adomian polynomials which is defined by:

$$A_n = U_n \frac{\partial U_n}{\partial x} \tag{9}$$

The scalar multiplication of the inverse operator and the Adomian polynomial results to a recursive relation been expressed as

$$U_{n+1} = -L^{-1}(A_n) \tag{10}$$

where L^{-1} is the inverse operator written as $\int_0^t A_n d\tau$

hence
$$U_{n+1} = \int_0^t A_n d\tau \tag{11}$$

Convergence Analysis

- Assumptions for convergence of VIM

Under suitable conditions, the sequence converges to the exact solution, taking the Limit of U_n as n tends to infinity then we have

$$\lim_{n \rightarrow \infty} U_n = U \tag{12}$$

VIM often exhibits rapid convergence due to error correction in each iteration. The convergence of VIM depends on:

- a. Proper identification of Lagrange multipliers say $\lambda(t)$
- b. Smoothness of the nonlinear operator.

- Assumptions for convergence of ADM

The convergence of ADM given by (12) depends on both the convergence of the series and behavior of Adomian polynomials. $\sum U_n$ (13)

Thus, (13) must satisfy the following sufficient conditions

- a. L is Lipschitz continuous, that is, $\|L(U) - L(V)\| \leq L\|U - V\|, \forall U, V \in X$, for a Banach space X and some constant $L > 0$.
- b. Assume an initial approximation $U(x, t)$ with sufficient regularity in X
- c. The operator $T : X \rightarrow X$ is bounded and hence continuous, i.e, there exists $M > 0$ such that $\|T(U)\| \leq \|M(U)\|$, for all $U \in X$

Error Analysis

The error analysis used in this paper is given as

$$\sum^n (\text{error}) = | U(x, t) - U_n(x, t) | \tag{14}$$

where $U(x, t)$ is the exact solution, and $U_n(x, t)$ is the approximation obtained at the n th iteration. VIM error decreases rapidly due to iterative correction and ADM error depends on truncation of the series.

Comparative Convergence Analysis

Features	VIM	ADM
Convergence speed	Fast	Moderate
Computational complexity	Low	Moderate
Handling non linearity	Direct	Requires polynomias
Accuracy	High	High
Implementation	Simple	Slightly complex

Numerical Illustration

For the governing Equation, consider the nonlinear gas dynamic (Burgers-type) equation as in (1)

$$\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} = v \frac{\partial^2 u}{\partial x^2}$$

with initial condition: $U(x, 0) = \sin(x)$

The numerical simulations presented below are used to guide the iterative scheme.

With Domain: $x \in [0, 2\pi]$; Time: $t \in [0, 1]$; Viscosity: $V = 0.1$ and

Iterations: 5–10 terms for both VIM and ADM

Numerical Iterations and Results

The Iterations for sine and cosine are in Radians

The values of x are selected uniformly over the interval: $0 \leq x \leq 2\pi$, such that $2\pi = 6.283$ using

$$\Delta x = (2\pi/N)t ; t = 0, 1, 2, \dots, 9 \text{ and } N = 9 \tag{15}$$

Then we have the following Iterations;

When $t = 0, x = (6.283/9) \times 0 = 0$

$t = 1; x = (6.283/9) \times 1 = 0.698$

$$t = 2; x = (6.283/9) \times 2 = 1.396$$

$$t = 3; x = (6.283/9) \times 3 = 2.094$$

Thus $x_i = 0, 0.698, 1.396, 2.094, \dots, 6.283$

Exact solution U(x, t) first, second and third iterations

Given the exact solution as $u(x, t) = \sin(x)$, from $t = 0, 1, 2, 3, \dots, 9$ and $x = 0, 0.698, 1.396, \dots, 6.283$

Iterations

At $t = 0, x = 0$; then $U(x, t) = \sin(x) = \sin(0) = 0.0000$

At $t = 1, x = 0.698$; then $U(x, t) = \sin(0.698) = 0.6427$

At $t = 2, x = 1.396$; then $U(x, t) = \sin(1.396) = 0.9848$, and so on.

For the 10 iterations, we have the table below

Table 1: The Values of x and the exact solution U(x, t)

t	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
X	0.000	0.698	1.396	2.094	2.793	3.491	4.189	4.889	5.585	6.283
Initial U(x, t)	0.0000	0.6428	0.9848	0.8660	0.3420	-0.3420	-0.8660	-0.9848	-0.6428	0.0000

VIM Approximate Solution (First and second Iterations)

From (4), we have the governing equation below in the form of homogeneous equation when $V = 0$

$$\frac{\partial u}{\partial \tau} + U \frac{\partial u}{\partial x} = 0 \tag{16}$$

Recall from (6) we applied VIM to the Nonlinear gas dynamics equation, hence we have the VIM correction functional

$$U_{n+1} = U_n + \int_0^t \lambda \left(\frac{\partial U_n}{\partial \tau} + U_n \frac{\partial U_n}{\partial x} - v \frac{\partial^2 U_n}{\partial x^2} \right) d\tau \tag{17}$$

The Algorithm for VIM Iterations are as follows:

Step 1: Start with an initial approximation given in (18) below, which may be an initial guess or an exact solution to a simpler form of the problem, and apply the iterative process to improve the approximation.

Repeat this process until the approximation converges to a desired accuracy

$$U(x, t) = \sin(x) \tag{18}$$

Step 2: First iteration $u_1 = u_1 - \int_0^t (u_0 u_{0x}) d\tau$ (19)

Step 3: Compute derivatives differentiating (18) we have $u_{0x} = \cos(x)$ (20)

Combine with (18) with $U(x, 0)$ we have

$$U_0 U_{0x} = \sin(x) \cos(x) \tag{21}$$

Step 4: Integrating : $u_{VIM} = \sin(x) - t \sin(x) \cos(x)$ (22)

Step 5: Monitor the convergences of the Iterations process by Substituting for $t = 0.5$ a such we have the VIM approximation model below

$$u_{VIM}(x, 0.5) = \sin(x) - 0.5 \sin(x) \cos(x) \tag{23}$$

Using (23) with $x = 0, 0.698, 1.396, \dots, 6.283$, we have the following Iterations

At $t = 0$; then $\sin(0) = 0, \cos(0) = 1, u_{VIM} = 0 - 0.5(0 \times 1) = 0$

At $x = 0.698, \sin(0.698) = 0.6428, \cos(0.698) = 0.7660,$

$$u_{VIM} = 0.6428 - 0.5(0.6428 \times 0.7660), u_{VIM} = 0.3966$$

At $x = 1.396, \sin(1.396) = 0.9848, \cos(1.396) = 0.1739,$

$$= 0.9848 - 0.5(0.9848 \times 0.1739) = 0.8992$$

Hence Table 2 below

Table 2: VIM Approximation at $t = 0.5$

t	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
X	0.000	0.698	1.396	2.094	2.793	3.491	4.189	4.889	5.585	6.283
Initial U(x, t)	0.0000	0.6428	0.9848	0.8660	0.3420	-0.3420	-0.8660	-0.9848	-0.6428	0.0000
VIM approx	0.0000	0.3966	0.8992	1.0825	0.5027	-0.5027	-1.0825	-0.8992	-0.3966	0.0000

ADM Approximate Solution

Recall from (8), $U_n = \sum_{n=0}^{\infty} U_n$ (24)

with the initial terms given as $u_0 = \sin(x)$ (25)

from (8) the nonlinear term decomposed using Adomian polynomials,

we have $UU_n = \sum_{n=0}^{\infty} A_n$ (26)

differentiating (25) and multiply the result with (25)

we have the Adomian polynomials as $A_0 = U_0 U_{0x} = \sin(x)\cos(x)$ (27)

from (11) the Recurrence relation is $U_{n+1} = - \int_0^t A_n d\tau$ **

put (27) into (**) to get $u_1 = -t \sin(x) \cos(x)$ (28)

and $u_2 = \frac{t^2}{2} \sin(x) \cos^2(x)$ (29)

The approximate solution is the sum of the initial terms (25), Recurrence relation i.e (28) and (29), hence we have the following $u_{ADM} = u_0 + u_1 + u_2$ (30)

And $u_{ADM} = \sin(x) - t \sin(x)\cos(x) + \frac{t^2}{2} \sin(x) \cos^2(x)$ (31)

Substituting for $t = 0.5$, we have the ADM approx model below

$u_{ADM}(x, t) = \sin(x) - 0.5\sin(x) \cos(x) + 0.125\sin(x)\cos^2(x)$ (32)

When $x = 0$, $\sin(0) = 0$, $\cos(0) = 1$ and

$u_{ADM} = 0 - 0.5(0 \times 1) + 0.125(0 \times 1^2) = 0$

When $x = 0.698$, $\sin(0.698) = 0.6428$, $\cos(0.698) = 0.7660$ and

$u_{ADM} = 0.6428 - 0.5(0.6428 \times 0.7660) + 0.125(0.6428 \times 0.7660^2) = 0.4438$

when $x = 1.396$, $\sin(1.396) = 0.9848$, $\cos(1.396) = 0.1739$

$= 0.9848 - 0.5(0.9848 \times 0.1739) + 0.125(0.9848 \times 0.1739^2), = 0.9029$

The table below shows for 10 iterations

Table 3: ADM Approximation at t = 0.5

t	0.0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
X	0.000	0.698	1.396	2.094	2.793	3.491	4.189	4.889	5.585	6.283
Initial U(x, t)	0.0000	0.6428	0.9848	0.8660	0.3420	-0.3420	-0.8660	-0.9848	-0.6428	0.0000
ADM approx	0.0000	0.4438	0.9029	1.1096	0.5404	-0.5404	-1.1096	-0.9029	-0.4438	0.0000

10. Convergence Analysis and Error

Table 4: Convergence rate of VIM and ADM Approximation

t	x	Initial U(x, t)	VIM approx	ADM approx
0.0	0.000	0.0000	0.0000	0.0000
1.0	0.698	0.6428	0.3966	0.4438
2.0	1.396	0.9848	0.8992	0.9029
3.0	2.094	0.8660	1.0825	1.1096
4.0	2.793	0.3420	0.5027	0.5404
5.0	3.491	-0.3420	-0.5027	-0.5404
6.0	4.189	-0.8660	-1.0825	-1.1096
7.0	4.887	-0.9848	-0.8992	-0.9029
8.0	5.585	-0.6428	-0.3966	-0.4438
9.0	6.283	0.0000	0.0000	0.0000

Error Analysis

Using the expressions VIM error = | VIM approx - u(x, t) |

And ADM error = | ADM approx - u(x, t) |

We obtain Table 5.

Table 5: Error Analysis of VIM and ADM Approximation

t	x	Initial U(x, t)	VIM approx	ADM approx	Error (VIM approx)	Error (ADM approx)
0.0	0.000	0.0000	0.0000	0.0000	0.0000	0.0000
1.0	0.698	0.6428	0.3966	0.4438	0.2462	0.1990
2.0	1.396	0.9848	0.8992	0.9029	0.0856	0.0819
3.0	2.094	0.8660	1.0825	1.1096	0.2165	0.2436
4.0	2.793	0.3420	0.5027	0.5404	0.1607	0.1984
5.0	3.491	-0.3420	-0.5027	-0.5404	0.1607	0.1984
6.0	4.189	-0.8660	-1.0825	-1.1096	0.2165	0.2436
7.0	4.887	-0.9848	-0.8992	-0.9029	0.0856	0.0819
8.0	5.585	-0.6428	-0.3966	-0.4438	0.2462	0.1990
9.0	6.283	0.0000	0.0000	0.0000	0.0000	0.0000

Graphical Analyses of Solution Profiles

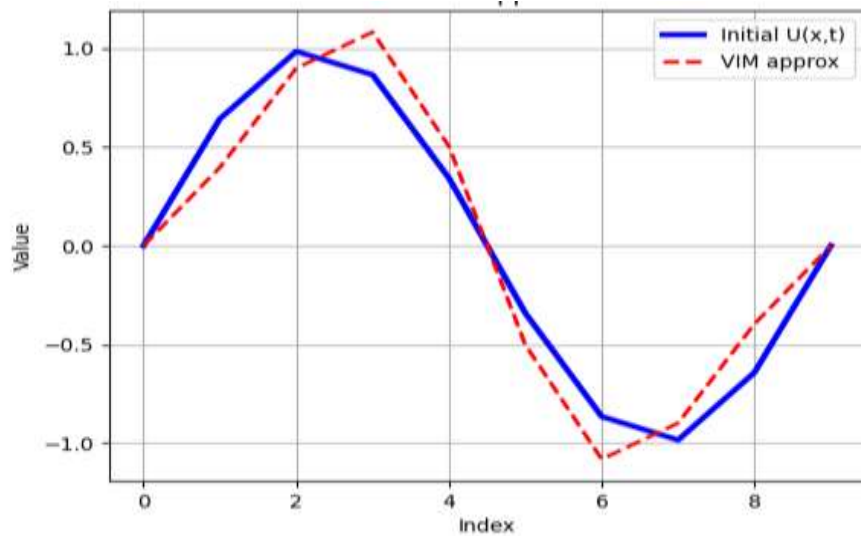


Figure 1: Solution Profile Comparison of $u(x, t)$ at $t=0.5$ with VIM approx.

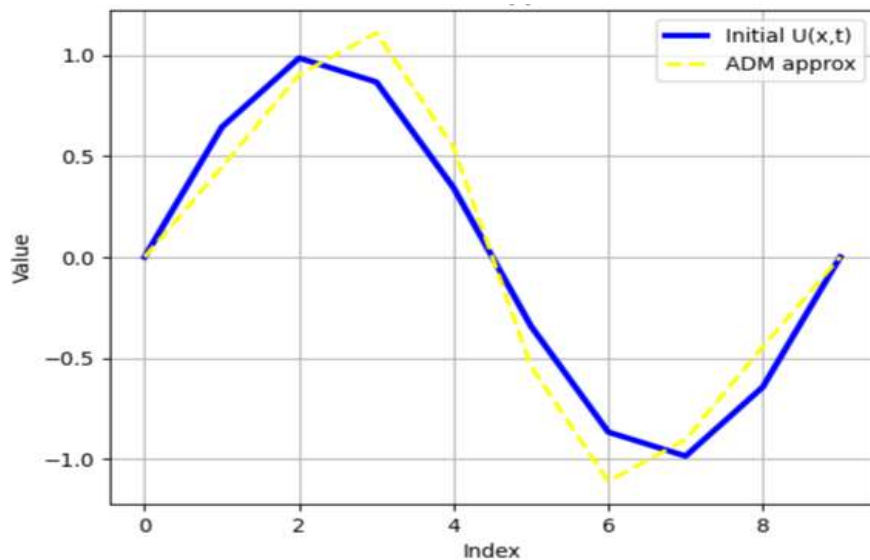


Figure 2: Solution Profile Comparison of $u(x, t)$ at $t=0.5$ with ADM approx.

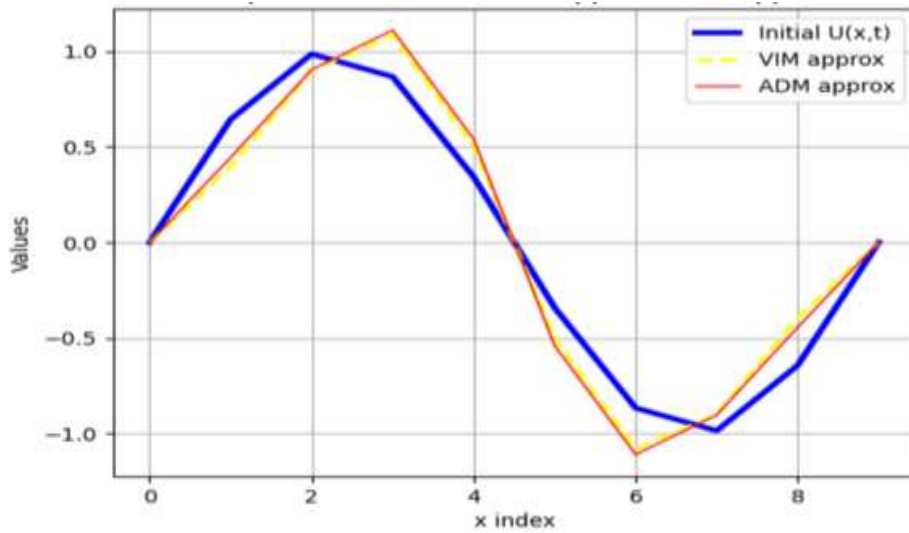


Figure 3: Convergence rate of VIM and ADM

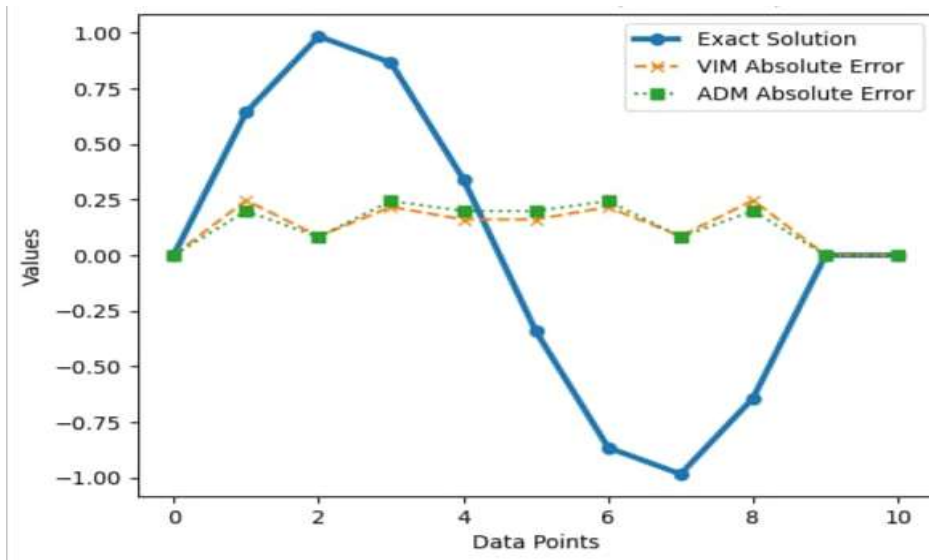


Figure 4: Absolute error distribution (VIM vs ADM vs exact)

DISCUSSION

Table 1 presents the summary of the x - values and the exact solution of 10 iterations. Table 2 presents the numerical values of the approximate solution obtained using the Variational Iteration Method (VIM) at $t=0.5$, figure 1 illustrated the comparison between the initial condition and the VIM approximation. It is observed that the solution maintains its wave-like structure while exhibiting nonlinear deformation. The results confirmed the stability and convergence efficiency of VIM. Table 3 shows the numerical output of the approximate solution obtained using the Adomian Decomposition Method (ADM) at $t = 0.5$. Figure 2 displays the comparison between the exact solution and the ADM approximation. Table 4 shows the convergence rate of the approximate value of the Variation iteration method (VIM) and the Adomian Decomposition Method (ADM) against the exact solution. Figure 3 show that both methods converge with VIM exhibiting fewer iteration compel to ADM.

Table 5 presents the comparison of the absolute errors for the Variational Iteration Method (VIM) and Adomian Decomposition Method (ADM) against a reference exact solution. It is observed that both methods produce relatively small errors across the domain. However, VIM demonstrates slightly better accuracy in regions near the peak of the solution, while ADM performs comparably in mid-range values. The error distribution confirms that both methods converge, with VIM showing a marginal advantage in convergence speed. Figure 4 show VIM reaches accurate approximation within fewer iterations, ADM produces smooth series solutions but requires more terms. Both methods converge to similar solutions as iteration increases.

This comparative study has shown that both methods are powerful tools for solving nonlinear gas dynamic equations. However the results indicate that:

- ✓ VIM is more efficient in terms of convergence speed.
- ✓ ADM provides a more structured analytical framework.
- ✓ Both methods yield highly accurate approximations under suitable conditions.

Future research may focus on hybrid techniques combining the strengths of both methods.

CONCLUSION

This study presents a comparative convergence analysis of the Variational Iteration Method (VIM) and the Adomian Decomposition Method (ADM) applied to nonlinear gas dynamic equations. Both methods provide efficient analytical approximations without linearization, but the VIM demonstrates faster convergence in most cases with fewer iterations. The ADM, while slightly slower, produces highly accurate results that closely match the exact solutions. Numerical simulations and error analysis confirm the reliability and effectiveness of both methods, making them valuable tools for solving complex nonlinear gas dynamics problems. Overall, the choice between VIM and ADM may depend on the required balance between computational efficiency and accuracy. This study has presented a detailed comparative convergence analysis of the Variational Iteration Method and the Adomian Decomposition Method applied to nonlinear gas dynamic equations.

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