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Predicting Extreme Persistence in Returns and Volatility of the Nigerian All Share Index using FIGARCH and ARFIMA-Based Models

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ABSTRACT
The study explores the existence of significant persistency in both returns and volatility of the Nigerian All Share Index (ASI) by employing sophisticated long-memory models, namely FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH frameworks. The analysis is based on a total of 3,138 daily data points collected from January 30, 2012, to February 7, 2024, corresponding to the trading days of the Nigerian Exchange. Initial evaluations conducted with the ARCH LM test indicate notable heteroskedasticity across several lags ($p < 0.05$), supporting the use of volatility-oriented modeling. A deeper investigation into long memory characteristics uncovers exceptionally pronounced persistence, with the Hurst exponent ($H \approx 1.13$) and the ARFIMA fractional differencing parameter ($d \approx 0.999$) reflecting near-unit-root behavior and hyper-persistent dynamics in the data series. The findings demonstrate that the FIGARCH model effectively captures a strong long-term dependence in volatility with a fractional integration coefficient ($D = 1.0$), while the ARFIMA–GARCH model detects moderate long memory in returns ($d \approx 0.06$) and significant volatility persistence. It is particularly noteworthy that the hybrid ARFIMA–FIGARCH model shows enhanced efficacy, with all parameters proving to be statistically significant, and diagnostic assessments affirming the model's adequacy, stability, and the non-existence of residual autocorrelation or heteroskedasticity. When comparing forecasts, it is observed that the FIGARCH model leads to a stable mean trajectory, while the ARFIMA-based models present curvilinear trends, aptly representing long-memory behavior in returns. Moreover, volatility forecasts generated by all models exhibit persistently mean-reverting tendencies, along with escalating uncertainty as the forecast horizon extends. The key novelty of this research is the identification of extreme (near-unity) long memory ($d \approx 0.999$) within the Nigerian stock market, which is remarkably greater than commonly observed figures (0–0.4) in both African and global financial markets, pointing towards hyper-persistent dynamics in returns and volatility. This serves as compelling evidence against the weak form of market efficiency and indicates that disturbances to the ASI have prolonged and gradually diminishing impacts. Additionally, the research makes methodological advancements by illustrating that hybrid ARFIMA–FIGARCH models which surpasses the traditional or singular long-memory models in capturing persistence in both mean and volatility aspects collectively. The implications of these results are significant for investors, policymakers, and financial analysts, underscoring the necessity for improved risk management techniques, enhanced market regulation, and the integration of advanced forecasting methods in emerging markets. The study enriches the field of financial econometrics by delivering fresh empirical insights into dual long-memory behavior and extreme persistence in the Nigerian stock market.
Keywords: Predicting, Extreme, Persistence, Returns, Volatility & All Share Index

1.0 INTRODUCTION
The behavours of stock market returns, and their volatility are significant concerns within financial econometrics, especially in developing markets like Nigeria. The Nigerian All Share Index is a vital measure of the market's overall performance, indicating investor attitudes, economic conditions, and

shifts in policy (Deebom et al, 2023). Nonetheless, the stock market in Nigeria frequently exhibits structural disruptions, policy-related shocks, and inefficiencies in information (Adewole, 2024). These factors complicate the prediction of both returns and volatility. Financial markets in emerging nations, such as Nigeria, often display a "sticky" behavior: shocks tend to have prolonged effects, and volatility tends to persist (Deebom et al, 2023). It is essential to accurately recognize this significant persistence in both returns and volatility for effective risk management, accurate pricing, and the regulation of the Nigerian All Share Index (NASI) (Deebom et al, 2023).

Conventional time series methods like ARIMA and GARCH assume of short memory processes, where the effects of shocks dissipate quickly (Dufitinema, 2021). However, financial time series often show long memory characteristics, where the impacts of shocks endure for longer periods and reduce gradually (Tuaneh, Deebom & Akah, 2025). This aspect suggests that historical information has a continuing effect on current and future market behaviors, thereby questioning the principles of market efficiency and randomness (Subair & Arewa, 2020). Long memory models like ARFIMA and FIGARCH are specifically created to address this type of behavior (Tripathy, 2022). Long memory signifies that the effects of shocks on returns or volatility diminish slowly, meaning that past information continues to impact future market activities over substantial timeframes (Subair & Arewa, 2020). This situation challenges the weak form of the Efficient Market Hypothesis, which posits that historical price data should not provide insights into future returns (Deebom et al., 2023). Evidence from financial markets, including Nigeria, indicates that returns and volatility can demonstrate such persistence, especially during times of economic instability (Tripathy, 2022).

To overcome these challenges, sophisticated models like the Autoregressive Fractionally Integrated Moving Average (ARFIMA) and the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity (FIGARCH) have been introduced. The ARFIMA model is designed to capture long memory within the mean (returns), while FIGARCH focuses on long memory in volatility. When these models are integrated into a hybrid ARFIMA-FIGARCH framework, they facilitate the simultaneous analysis of dual persistence in both returns and volatility, enhancing predictive accuracy and providing deeper insights into market behavior. Research indicates that ARFIMA-FIGARCH models surpass traditional models in accurately representing long-range dependencies and volatility clustering within financial time series (Odonkor, et al 2019). ARFIMA addresses long memory in the mean (returns), whereas FIGARCH reflects persistence in volatility. The integration into a hybrid ARFIMA-FIGARCH framework delivers a more versatile architecture that can capture simultaneous long memory in both returns and volatility, thereby boosting forecasting accuracy (Odonkor, et al, 2019).

Notwithstanding these developments, a deficiency exists in the existing literature concerning the forecasting of extreme persistence within the Nigerian stock market, especially using hybrid long-memory models. Consequently, this research aims to address this deficiency by utilizing ARFIMA–FIGARCH models on the Nigerian All Share Index, with the objective of boosting prediction accuracy and deepening insights into volatility patterns in the context of an emerging market

2.0 LITERATURE REVIEW

2.1 Theoretical Review

The basis of long-memory processes arises from the notion of fractional integration, a concept put forth by Clive Granger and Roselyne Joyeux, which was later built upon by Jonathan Hosking (Hosking, 1981). According to Deebom et al (2021) Long-memory processes characterize time series exhibiting slow decay in autocorrelations. The ARFIMA model builds upon the conventional ARIMA framework by integrating a fractional differencing factor, which enables it to accommodate persistence in return. In a similar fashion, the FIGARCH model enhances GARCH by incorporating fractional integration within the volatility aspect, which allows for the modeling of enduring volatility shocks. Such models offer a more adaptable approach for investigating financial time series that display long-range dependence (Subair & Arewa, 2020). These researchers (Clive Granger, Roselyne Joyeux, and Jonathan Hosking) illustrated that time series can show persistence that exists between stationary and non-stationary phenomena, facilitating the creation of the ARFIMA model. Likewise, the persistence of volatility is rooted in ARCH/GARCH theory, which later evolved into FIGARCH to permit fractional differencing in the variance equation, thereby capturing long-range dependence in volatility (Subair & Arewa, 2020).

2.2 Conceptual Review

In terms of conceptual foundations, the research hinges on three primary principles: market efficiency, volatility clustering, and long memory. The concept of market efficiency posits that prices completely reflect all available information; however, the presence of long memory indicates that returns and volatility exhibit predictability. Volatility clustering, where substantial changes follow hefty fluctuations, is a typical characteristic of financial data and is adeptly modeled by GARCH-type frameworks (Deebom et al., 2023). The combination of ARFIMA and FIGARCH creates a thorough framework that simultaneously represents persistence in both mean and variance processes (Deebom et al., 2023). Long memory is characterized by gradually diminishing autocorrelations, signifying that historical values exert influence on the present well into the future (Baillie, 1996). In conceptual terms, long memory elucidates a condition in which the autocorrelation function declines slowly, suggesting that shocks have extended consequences. Within financial markets, this is evident as persistent patterns in returns (predictability of the mean) and volatility clustering alongside persistence (predictability of the variance). While ARFIMA models encapsulate fractional integration in the mean process, FIGARCH expands GARCH to allow for fractional integration regarding volatility, effectively modeling ongoing shocks to conditional variance (Baillie, 1996; Tripathy, 2022; Maheshchandra, 2012). Research covering stocks, currencies, and macroeconomic data indicates that long-memory models frequently outperform short memory ARMA/GARCH in representing enduring dynamics (Tripathy, 2022; Adewole, 2024; Umoru et al., 2024; Dufitinema, 2021). The ARFIMA–FIGARCH framework combines these two aspects, facilitating dual persistence, which is especially pertinent in emerging markets characterized by inefficiencies and sluggish information diffusion.

2.3 Empirical Review

Empirical investigations across both global and emerging markets furnish substantial evidence of long memory characteristics in financial time series. For example, in the case of the Nigerian equity market: the ARFIMA, FIGARCH, and HYGARCH models uncover long-run dependencies in Nigerian stocks and advocate for the rejection of market efficiency (Subair & Arewa, 2020; Deebom et al., 2021; Deebom et al., 2023). In relation to the Nigerian All Share Index, ARFIMA has been utilized for returns but has not incorporated a long memory volatility aspect like FIGARCH (Deebom et al., 2023); previous studies concerning the All-Share Index limited their focus to GARCH/EGARCH type frameworks (Iyiegbuniwe et al., 2012).

In the markets of Central and Eastern Europe, research indicated that the ARFIMA and FIGARCH models demonstrated substantial long memory in both returns and volatility, with the ARFIMA–FIGARCH combination yielding better forecasting accuracy. Likewise, analyses of the stock market in Pakistan revealed that although returns do not consistently show long memory, volatility consistently reflects enduring characteristics, indicating the validity of FIGARCH-type models.

Despite these insights, there are still notable gaps: Current research on Nigeria tends to concentrate on either ARFIMA or GARCH-type models in isolation, failing to merge both return and volatility dynamics into a cohesive model. A considerable number of studies highlight general long memory, yet they often overlook the significance of extreme persistence, which is essential for comprehending extended disturbances such as financial crises or instability in policy. There is a scarcity of empirical research that specifically examines the Nigerian All Share Index utilizing hybrid ARFIMA–FIGARCH models, particularly concerning their forecasting efficiency. Furthermore, many studies do not sufficiently connect long memory findings to practical implications for stakeholders, including investors, policymakers, and those involved in risk management. Consequently, this research seeks to fill these gaps by employing a hybrid ARFIMA–FIGARCH framework that captures long memory in both returns and volatility concurrently. The primary aim of this study is to delve into extreme persistence, offering enhanced understanding of sustained market disruptions and their consequences. By tailoring the model to the

Nigerian All Share Index, it presents localized and relevant evidence for policy considerations. This approach improves forecasting precision and delivers actionable insights for investment strategies, risk management, and the development of financial policies in Nigeria.

3.0 METHODOLOGY

3.1 Data Description and Source

This research made use of daily closing data on All-Share Index (ASI) extracted from the Nigerian Stock Exchange. The data set comprises of 3138 observations covering the period of January 30, 2012, to February 7, 2024, according to the trading days of the Nigerian Exchange. Before the data analysis was carried out, the data underwent multiple preprocessing steps. Initially, the dataset was examined for any missing values. Subsequently, a z-score analysis was performed using a rolling data window encompassing 180 data points.

3.2 Exploratory Data Analysis

Following this, the Augmented Dickey-Fuller (ADF) test was conducted to evaluate the stationarity of the time series. To address non-stationarity in the series, first-order differencing was applied, as indicated by the outcomes of the ADF test. Similarly, time plots were generated for raw ASI series, moving averages and Log returns. The Log returns: $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)X100$

These plots provide visual insights into trends, seasonality, and volatility clustering. Also, the structural breaks were examined using statistical tests such as the Bai–Perron multiple breakpoint test: $y_t = x_t'\beta_j + u_t \text{for } t = T_{j-1} + 1, \dots, T_j$

Breakpoints were identified and corresponding periods were documented to capture regime shifts in the series. In another development, descriptive statistics were computed for both raw and return series to test for normality using the Jarque Bera test statistics given as: $JB = \frac{n}{6}\left(S^2 + \frac{(K-3)^2}{4}\right)$

where $S - \text{Skewness}$, $K - \text{Kurtosis}$, and $JB - \text{Jarque} - \text{Bera}$, $n - Number\ of\ Observations$. To determine the integration properties of the series, the following unit root tests were conducted, and they include Augmented Dickey-Fuller (ADF) Test, Phillips-Perron (PP) Test and KPSS Test. The Augmented Dickey-Fuller (ADF) Test is given as: $\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \delta_i \Delta y_{t-i} + \epsilon_t$

In another development, the Phillips-Perron (PP) Test for stationarity was also considered. This was done to adjust serial correlation and heteroskedasticity without adding lagged differences (Deebom &Essi,2017). In a similar manner, the KPSS test was utilized to analyze the stability of the time series, assuming the null hypothesis of stability. Initially, the series was regressed against a deterministic factor (either constant or constant with a trend), and the residuals generated from this regression were utilized to calculate the partial sum process. Subsequently, the KPSS statistics were computed by taking the ratio of the total sum of squared cumulative residuals to the assessed long-term variance. The analysis was performed at both the original levels and the first differences to ascertain the order of integration of the series. The KPSS test was utilized to analyze the stability of the time series, assuming the null hypothesis of stability. Initially, the series was regressed against a deterministic factor (either constant or constant with a trend), and the residuals generated from this regression were utilized to calculate the partial sum process. Subsequently, the KPSS statistics were computed by taking the ratio of the total sum of squared cumulative residuals to the assessed long-term variance. The analysis was performed at both the original levels and the first differences to ascertain the order of integration of the series. The foundation of the KPSS test is based on the equation:

$$y_t = \delta t + \mu_t + \varepsilon_t,\ \mu_t = \mu_{t-1} + u_t$$

where: y_t is observed time series, δt is deterministic trend (included in trend-stationarity case), μ_t is random walk (non-stationary component), ε_t is stationary error term, u_t is white noise and ϵ_t is the stationary error term and $u_t \sim iid(0, \sigma_u^2)$. The KPSS test assumes stationarity under the null hypothesis, unlike tests like Augmented Dickey-Fuller test. The KPSS test statistics are given as: $KPSS = \frac{1}{T^2}\sum_{t=1}^{T} S_t^2/\hat{\sigma}^2$. where: $S_t = \sum_{i=1}^{t}\hat{\epsilon}_i$ (partial sum of residuals), $\hat{\epsilon}_t$ = residuals from regression and $\hat{\sigma}^2$ = long-run variance estimate , The hypotheses are given as $\mathrm{H_0}$: The series is stationary versus $\mathrm{H_1}$: The series is non-stationary. It is used to test whether: $\sigma_u^2 = 0 \rightarrow$ stationary and $\sigma_u^2 > 0 \rightarrow$ unit root (non-stationary). The decision rule is that we reject H_0 if KPSS statistic than the critical value and fail to reject H_0 if KPSS statistic ≤ critical value. Additionally, the ACF and PACF graphs were employed to determine the suitable lag arrangement for the ARIMA model. The trends found in the correlograms assisted in the initial choice of autoregressive and moving average components, which were later optimized using the Akaike Information Criterion to achieve the most economical model. The sample Autocorrelation Function (ACF) at lag k is defined as:

$$\rho_k = \frac{\sum_{t=k+1}^{n}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{n}(y_t - \bar{y})^2}$$

where y_t is the observed time series, $\bar{y}$ is the sample mean and k is the lag length. The ACF measures the correlation between observations separated by k periods. The partial autocorrelation on the other hands at lag k is denoted ϕ_{kk}, measures the direct relationship between y_t and y_{t-k}, controlling for intermediate lags. It is obtained from the following Autoregressive equation is represented as: $y_t = \phi_{k1}y_{t-1} + \phi_{k2}y_{t-2} + \cdots + \phi_{kk}y_{t-k} + \epsilon_t$

The Partial Autoregressive Function (PACF) at lag is represented by the coefficient ϕ_{kk}. The identification of ARIMA pdq models adhere to established patterns, including that in an AR(p) process, the PACF is truncated after lag p while the ACF decreases gradually. Likewise, in an MA(q) process, the ACF is truncated after lag q, with the PACF gradually decreasing. For the ARMA(p,q) process, both ACF and PACF exhibit a decline. Additionally, the examination of long memory in the series employed a blend of Rescaled Range analysis, estimation of the Hurst exponent, the Geweke and Porter-Hudak method, and Lo's modified R/S test. These methodologies offer supportive evidence regarding the persistence characteristics of the series. The statistical significance of the fractional differencing parameter further validates if the series displays long-range dependence, thus supporting the use of ARFIMA and FIGARCH-type models.

3.3 Test for the Presence of Long Memory

The following tests for the presence of long memory were conducted, and they include hurst exponents, Geweke and Porter-Hudak (GPH) estimator, Rescaled Range (R/S) Analysis, Lo's Modified R/S Test and model-based fractional integration.

3.3.1 Hurst Exponent

The estimation of the Hurst exponent utilized the rescaled range (R/S) method. The time series was segmented into sub-periods of different lengths, and the rescaled range statistic $R(n)/S(n)$was calculated for each sub-period. The logarithm of the rescaled range was then regressed against the logarithm of the associated time span. The slope of this regression serves as an estimate for the Hurst exponent H, indicating the extent of long-range dependence present in the series. The Hurst exponent relies on the scaling relationship: $E\left[\frac{R(n)}{S(n)}\right] = Cn^H$

Taking logarithms:$\log\left(\frac{R(n)}{S(n)}\right) = \log C + H\log(n)$. The estimation equation (regression form)$\log(R/S)_n = \alpha + H\log(n) + \varepsilon_n$; where:$R(n)$ is range of cumulative deviations, $S(n)$represents standard deviation , n is the sample size or sub-period length. H is the Hurst exponent, α *is the* $\log C$ and $\varepsilon_n$is error term. By interpretations when H = 0.5, there is no memory (random walk), H > 0.5 meaning long memory (persistence) and H < 0.5 represents anti-persistence.

3.3.2 Geweke and Porter-Hudak (GPH) Estimator

The GPH estimator is a semi-parametric method used to estimate the fractional differencing parameter d based on the spectral density of the series at low frequencies. The model is specified as: $\log I(\lambda_j) = \beta_0 - d\log(4\sin^2(\lambda_j/2)) + u_j$

where:$I(\lambda_j)$= periodogram of the series at frequency λ_j, $\lambda_j = \frac{2\pi j}{n}, j = 1,2,\dots,m$(Fourier frequencies), d= fractional differencing parameter and u_j= error term. By estimation, the parameter d is estimated as the slope coefficient obtained from the regression of $\log I(\lambda_j)$on $\log\left(4\sin^2(\lambda_j/2)\right)$. To interpret the results, when d *is* 0(it means there is no long memory), when $0 < d < 0.5$(there is long memory and stationary persistence) and $d \geq 0.5$(non-stationary long memory).

3.3.3 Rescaled Range (R/S) Analysis

The Rescaled Range (R/S) statistics were employed to examine the persistence properties of the time series in the time domain. It is defined as:

$$\frac{R(n)}{S(n)} = \frac{\max\limits_{1\le k\le n}\left(\sum_{t=1}^{k}(X_t-\bar{X})\right) - \min\limits_{1\le k\le n}\left(\sum_{t=1}^{k}(X_t-\bar{X})\right)}{S(n)}$$

where X_t= time series, $\bar{X}$= sample mean $\bar{X} = \frac{1}{n}\sum_{t=1}^{n} X_t$. Also, $S(n)$= sample standard deviation such

that $S(n) = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(X_t-\bar{X})^2}$. On the other hands, the $R(n)$ is the range of cumulative

deviations and it is define as : $R(n) = \max\limits_{1\le k\le n} W_k - \min\limits_{1\le k\le n} W_k$ and W_k is the cumulative deviation and

this is define as; $W_k = \sum_{t=1}^{k}(X_t-\bar{X})$

3.3.4 Lo's Modified R/S Test

Lo (1991) modifies the R/S statistics to account for short-term dependence is given as:

$Q_n = \frac{R(n)}{\hat{S}_q(n)}$ while the adjusted standard deviation is given as

$$\hat{S}_q^2(n) = S^2(n) + 2\sum_{j=1}^{q} w_j(q)\gamma_j$$

where γ_j= sample autocovariance at lag j, $\gamma_j = \frac{1}{n}\sum_{t=j+1}^{n}(X_t-\bar{X})(X_{t-j}-\bar{X})$ and $w_j(q)$ is the weight function (Bartlett kernel) and $w_j(q) = 1 - \frac{j}{q+1}$; q= truncation lag . The final test statistics is given as: $V_n(q) = \frac{Q_n}{\sqrt{n}}$. However, the decision rule is reached when we compare $V_n(q)$ with critical values of 5% level of Significance for lower bound (0.809) and upper bound (1.862). However, for the purpose of interpretation if $V_n(q)$ lies within bounds, then there is no long memory whereas if outside bounds, then there is presence of long memory

3.3.5 Model-Based Fractional Integration

To investigate the existence of long memory in the dataset, a fractional integration technique based on a model was utilized. This method broadens the conventional differencing operator to accommodate integration orders that are not whole numbers. The model for fractional integration is defined as: $(1-L)^d y_t = \epsilon_t$

where L represents the lag operator such that $L^k y_t = y_{t-k}$, d stands for the fractional differencing parameter and $\epsilon_t \sim \text{i.i.d.}\ (0,\sigma^2)$ is the white noise error term. The operator $(1-L)^d$ can be expanded using the binomial series as:

$(1-L)^d = \sum_{k=0}^{\infty}\binom{d}{k}(-L)^k$ where: $\binom{d}{k} = \frac{d(d-1)(d-2)\cdots(d-k+1)}{k!}$

The parameter $d$captures the degree of persistence in the series. Specifically, when $d\ is\ 0$(short memory and a stationary process) , also when d lie between 0 and 0.5 ($0 < d < 0.5$) long memory and stationary with persistence existed and $d \geq 0.5$(non-stationary long memory process). A statistically significant positive value of $d$indicates the presence of long-range dependence in the series, thereby justifying the use of fractional models such as ARFIMA and FIGARCH.

3.4 Test for ARCH Effects

In order to identify conditional heteroskedasticity within the residuals of the mean equation, this research utilized the Autoregressive Conditional Heteroskedasticity (ARCH) Lagrange Multiplier (LM) test that was introduced by Engle(1982)(Deebom & Essi,2017). The test relies on the subsequent auxiliary regression:

$$\epsilon_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + u_t$$

where:$\epsilon_t$represents the residuals from the estimated mean equation, $\epsilon_t^2$is the squared residual, $\alpha_0$is a constant term, $\alpha_i$are the ARCH parameters, $q$is the number of lags and $u_t$is a white noise error term . The null hypothesis of no ARCH effect is given as $H_0: \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$ against the alternative H_1:At least one $\alpha_i \neq 0$. The test statistics are computed as: $LM = nR^2$. where n represents the sample size and $R^2$is the coefficient of determination from the auxiliary regression. The test statistics follow a chi-square distribution with $q$degrees of freedom. In this study, the ARCH LM test was conducted at lag lengths of 5, 10, and 15 to ensure robustness in detecting volatility clustering. Rejection of the null hypothesis indicates the presence of ARCH effects, thereby justifying the application of GARCH-type models for volatility modeling. The confirmation of long memory through fractional integration and the presence of ARCH effects provide strong empirical justification for the adoption of ARFIMA and FIGARCH-type models, which jointly capture persistence in both the mean and variance of the series.

3.5 Model Specification and Estimation

To capture both short-run dynamics and long-range dependence in the Nigerian Exchange Group All Share Index (ASI), the study employed a sequence of econometric models, beginning with the traditional ARIMA framework and extending to fractional and volatility-based models. This progression allows for a comprehensive modeling of both the mean and variance dynamics of the series. The baseline model employed in this study is the ARIMA (p, d, q)model, specified as: $\phi(L)(1-L)^d y_t = \theta(L)\epsilon_t$

where:$\phi(L) = 1 - \phi_1 L - \cdots - \phi_p L^p$is the autoregressive polynomial,

$\theta(L) = 1 + \theta_1 L + \cdots + \theta_q L^q$is the moving average polynomial, $d$is the order of integer differencing and $\epsilon_t \sim \text{i.i.d.}\ (0, \sigma^2)$ The ARIMA model captures short-run dependencies in the series and serves as the foundation for subsequent extensions. Autoregressive parameters (ϕ_i) measure the dependence of the current value of the series on its past values and its significant $\phi_i$indicates persistence in returns. Also, the high magnitude of the estimate suggests strong dependence on past observations. Also, the moving average parameters (**θ_j**) capture the impact of past shocks (innovations) on current values and the significance of the $\theta_j$indicates that past shocks influence current movements. It also helps correct for serial correlation in residuals. Model identification is guided by ACF and PACF plots, while parameter selection is based on the Akaike Information Criterion (AIC).

3.6 Model Specification

3.6.1 The ARFIMA (p, d, q)model:

To account for long memory in the mean process, the ARIMA model is extended to the ARFIMA (p, d, q)model: $\phi(L)(1-L)^d y_t = \theta(L)\epsilon_t$

where $d \in \mathbb{R}$is now a fractional differencing parameter. Unlike ARIMA, which assumes integer differencing, ARFIMA allows for fractional integration, thereby capturing persistence that decays at a hyperbolic rate. This makes it suitable for financial time series exhibiting long-range dependence. The ARFIMA model generalizes the ARIMA model by relaxing the integer differencing assumption. When $d$is restricted to integer values, ARFIMA reduces to the standard ARIMA model. Thus, ARIMA is a special case of ARFIMA.

3.6.2 Fractionally Integrated GARCH (FIGARCH) Model

The FIGARCH model, known as the Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity, was initially proposed by Clive, Granger and Tim Bollerslev in the year 1980 (Deebom et al, 2023), with additional enhancements made by Baillie Richard , Tim Bollerslev, and Mikkelsen in 1996 (Tuaneh, Deebom & Akah, 2025). This model operates on the concept of long memory in volatility, which improves the traditional GARCH structure by allowing shocks to diminish at a hyperbolic pace rather than an exponential one (Oummou et al. 2026). The foundational theory is derived from the concept of fractional integration in the analysis of time series, especially in relation to long-memory processes. The FIGARCH model extends the GARCH framework to incorporate long memory in volatility:

$$\sigma_t^2 = \omega + [1 - \beta(L)]^{-1}[1 - (1 - L)^d]\epsilon_t^2$$

where: $\sigma_t^2$is the conditional variance, $\omega > 0$is a constant, $\beta(L)$is the lag polynomial and $d$is the fractional differencing parameter in volatility. The FIGARCH model captures persistent volatility shocks, where the impact of past shocks decays slowly over time. The FIGARCH model generalizes the standard GARCH model by allowing the persistence parameter to be fractional. When $d = 0$, the FIGARCH model reduces to the standard GARCH model, implying short memory in volatility. While ARIMA and ARFIMA capture the dynamics of the mean equation, financial time series often exhibit volatility clustering, which requires modeling the conditional variance.

3.6.3: ARFIMA–FIGARCH Model

The Autoregressive Fractionally Integrated Moving Average–Generalized Autoregressive Conditional Heteroskedasticity (ARFIMA–GARCH) model merges the ARFIMA model brought forth by Granger and Roselyne Joyeux (1980) alongside Hosking's (1981) contribution with the GARCH model formulated by Tim Bollerslev (1986) (Deebom &Essi, (2017). The foundational theory unifies long memory within the mean process with conditional heteroskedasticity present in the variance process (Oummou et al. 2026). This model proves particularly beneficial in empirical finance settings where both return persistence and volatility clustering are present. Its widespread utilization is seen in the modeling of stock indices, macroeconomic indicators, and financial time series that display a gradual decline in autocorrelations along with fluctuating volatility rate (Oummou et al. 2026). The use of the ARFIMA–GARCH model is vital for enhancing forecasting precision, specifically in markets undergoing structural transitions and continual shocks. Within developing economies like Nigeria, where financial markets frequently exhibit long memory and clustering of volatility, this model offers a robust approach for risk evaluation and portfolio management. The normal form features a mean equation that incorporates fractional differencing to address long memory, along with a conventional GARCH (1,1) variance equation for modeling volatility clustering (Deebom & Essi,2017). The parameter for fractional integration facilitates the representation of ongoing dependence within the series, while the GARCH component captures short-term volatility behavior. The distribution of error is often assumed to be Student-t to account for leptokurtosis observed in financial returns**.** The ARFIMA–FIGARCH model combines long memory in the mean equation (ARFIMA) with long memory in the variance equation (FIGARCH), allowing for dual persistence in both returns and volatility. The mean equation is given is given as ARFIMA (p, d, q), therefore, the ARFIMA model captures fractional integration (long memory) in return is represented as:

$$\Phi(L)(1 - L)^d(y_t - \mu) = \Theta(L)\varepsilon_t$$

where: y_t represents the return series (e.g., ASI returns), μ stands for constant mean , $\varepsilon_t$is the error term with conditional variance σ_t^2, $\Phi(L)$is the autoregressive (AR) polynomial , $\Theta(L)$ represents the moving average (MA) polynomial, L stands for lag operator and d is the fractional differencing parameter ($-0.5 < d < 1$) while the expanded form of the ARFIMA(1,d,1) model is given as :

$$(1-\phi_1 L)(1-L)^d(y_t-\mu) = (1+\theta_1 L)\varepsilon_t$$

However, the Equation which captures the Variance is the FIGARCH (p, d, q) and so the FIGARCH model captures long memory in volatility which represented as:

$$\phi(L)(1-L)^D\varepsilon_t^2 = \omega + [1-\beta(L)]\sigma_t^2$$

A more common representation is:

$$\sigma_t^2 = \omega + [1-\beta(L)]^{-1}[1-\phi(L)(1-L)^D]\varepsilon_t^2$$

Where:σ_t^2 represents the conditional variance, ω stands for constant variance term, $\varepsilon_t^2$is the squared residuals, $\phi(L), \beta(L)$ are ARCH and GARCH lag polynomials respectively. While the D is the fractional integration parameter for volatility which lies between ($0 \leq D \leq 1$) The FIGARCH (1, D, 1) model is given as:

$$\sigma_t^2 = \omega + \beta_1\sigma_{t-1}^2 + [1-\beta_1 L-(1-\phi_1 L)(1-L)^D]\varepsilon_t^2$$

Error term assumption $\varepsilon_t = z_t\sigma_t, z_t \sim i.i.d.(0,1)$

Where $z_t$may follow normal distribution, or Student-t distribution (to capture heavy tails). Therefore, the

Complete Hybrid ARFIMA–FIGARCH Model

$$\Phi(L)(1-L)^d(y_t-\mu) = \Theta(L)\varepsilon_t$$
$$\sigma_t^2 = \omega + \beta(L)\sigma_{t-1}^2 + [1-\beta(L)-\phi(L)(1-L)^D]\varepsilon_t^2$$

some of the key features of the model are the dual long memory **(**d represents persistence in returns and the D is persistence in volatility). Also, the special cases of the model are**;** If $d = 0$: reduces to ARMA, if $D = 0$: reduces to GARCH and if $D = 1$: approaches IGARCH. By interpretation; when $d > 0$ long memory in mean (returns), $D > 0$: long memory in volatility and values of D close to 1 indicate extreme persistence. This combined framework allows for long memory in returns (mean equation via ARFIMA) and long memory in volatility (variance equation via FIGARCH). When $d\ is\ 0$, it shows that there is short memory volatility (GARCH) while $0 < d < 1$(Long memory volatility). When d is significant in the variance equation implies that volatility shocks persist for a long period, reflecting volatility clustering and slow decay of shocks.

In volatility models, ARCH parameter (α) measures the short-run impact of past shocks on current volatility. This reflects when α is high volatility reacts strongly to recent shocks, and it indicates news impact or market sensitivity where GARCH parameter (β) captures the persistence of past volatility. This reflects when β High volatility is said to be highly persistent, and this indicates volatility clustering. Also, volatility persistence is estimated using **($\alpha + \beta$)** and if $\alpha + \beta < 1$; then volatility means reverting, if $\alpha + \beta \approx 1$ then there are long-lasting shocks (high persistence). Conversely, if $\alpha + \beta > 1$, then the volatility process is explosive. The constant term (ω) represents the long-run average variance and it must be positive ($\omega > 0$). Higher ω shows a higher unconditional level of volatility.

In the combination of the ARFIMA–FIGARCH model, d(mean) represents persistence in returns, d(variance) shows persistence in volatility, $\alpha$is reaction to shocks and β is the persistence of volatility

3.5: Method of Estimation

All models were estimated using the Maximum Likelihood Estimation (MLE) technique.

The likelihood function is specified as: $L(\theta) = \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\sigma_t^2}} \exp(-\frac{\epsilon_t^2}{2\sigma_t^2})$ and the log-likelihood function: $\ln L(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left[\ln(2\pi) + \ln(\sigma_t^2) + \frac{\epsilon_t^2}{\sigma_t^2}\right]$

where $\theta$is the vector of parameters, $\epsilon_t$are residuals and $\sigma_t^2$is conditional variance. Parameter estimation was carried out by maximizing the log-likelihood function using numerical optimization techniques. For robustness, alternative error distributions such as the student-t distribution were considered to account for excess kurtosis in financial returns. The ARFIMA–FIGARCH model was ultimately adopted as the preferred specification due to its ability to simultaneously capture long memory in both the conditional mean and variance processes, thereby providing a more accurate representation of the underlying data-generating process.

4.0 RESULTS AND DISCUSSION

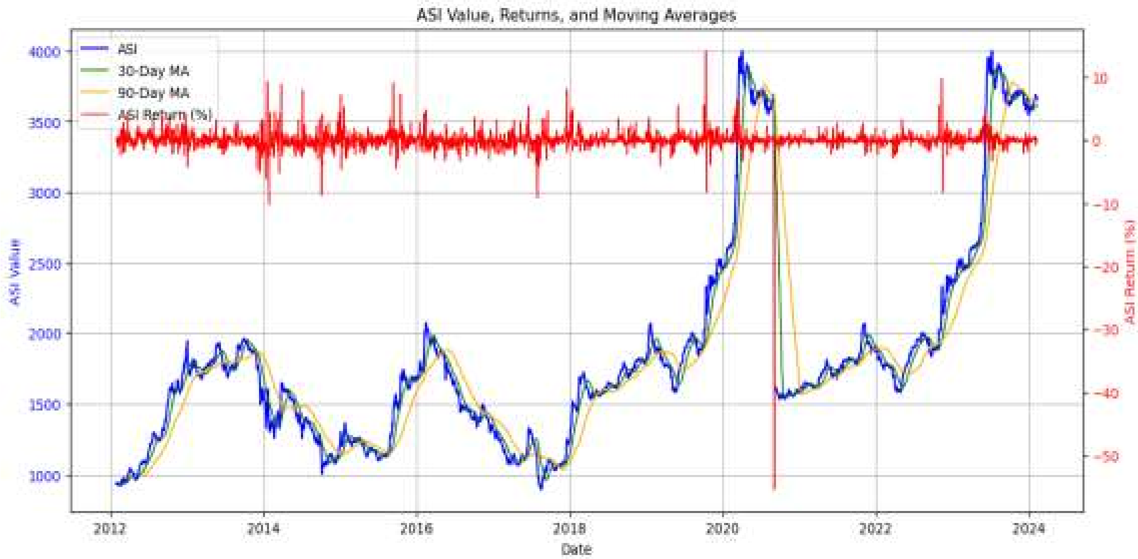

Figure 1: Time Plot on Observed Values on ASI, Returns and Moving Averages

Figure 1 displays a time plot of observed ASI series values along with their returns and moving averages. A visual assessment of this plot indicates significant variations over time, highlighting intervals with both increased and decreased activity, as well as possible clustering within the returns. The moving averages create a smoothing effect that reveals the underlying trend in the ASI data. This preliminary visualization supports evidence of non-stationarity within the series, which necessitates additional formal testing to identify the order of integration. The descriptive statistics of the ASI series are presented in Table 1 below were estimated to determine the mean, standard deviation, skewness, and kurtosis provide insight into the distributional properties of the data.

Table 1: Descriptive Statistics for Raw and Return series on ASI

	Raw Series	Returns
Mean	1836.431526	0.000434
Std Dev	721.847487	0.018748
Skewness	1.564999	-25.354112
Kurtosis	1.858745	1103.082521
Min	895.100000	-0.808077
Max	4000.790000	0.132801
JB p-value	0.000000	0.000000

The results of descriptive statistics for raw and return series on ASI are displayed in Table 1. The statistical descriptions reveal a mean return (0.000434) that is almost zero, paired with exceptionally high kurtosis (1103.0823) and negative skewness (-25.354), which points to a distribution of returns that is heavy-tailed and not normal. This situation suggests the occurrence of significant shocks, aligning with the characteristics of stock markets in Africa and other developing economies known for leptokurtic and asymmetric returns (Benbachir, 2025; Tripathy, 2022).

Table 2: Unit Root Tests (ADF, PP, & KPSS) for Raw series on ASI

Test	**Level p-value**	**First Difference p-value**	Remarks
ADF	0.682	0.000	Non-stationary at level, stationary at first difference
PP	0.553	0.000	Non-stationary at level, stationary at first difference
KPSS	0.010	0.100	Trend-stationary at first difference; non-stationary at level

The results of unit root for ASI are displayed in Table 1. Unit root tests including ADF, PP, and KPSS were conducted to examine the stationarity of the series. Tests for unit root indicate that the ASI level is non-stationary, but it becomes stationary following first (Magaji & Garba, 2023) differencing, which is a common characteristic of stock indices and macro-financial series that necessitate differencing or fractional differencing prior to modeling volatility (Adewole, 2024).

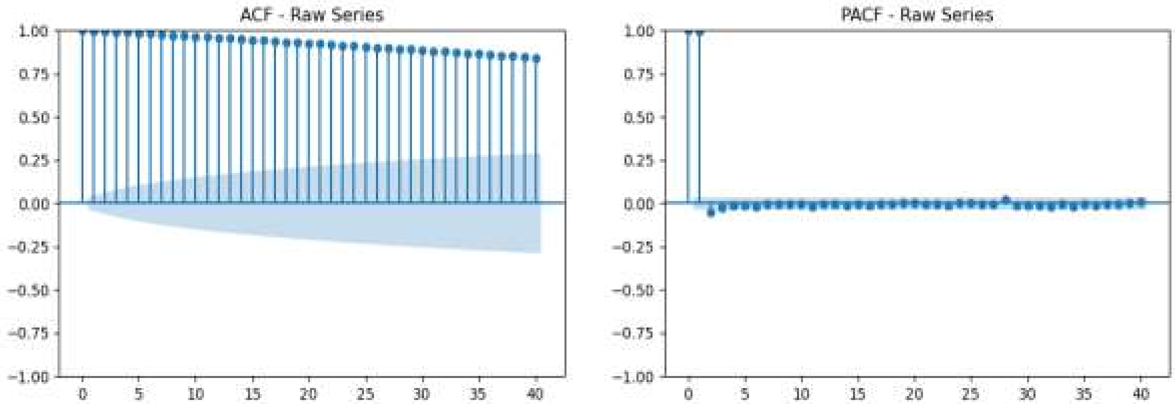

Figure 2: ACF and PACF Plots on Observed Values on ASI

Figure 3 illustrates the ACF and PACF plots for the ASI series. Noticeable spikes in both ACF and PACF at early lags indicate the presence of autocorrelation within the series, supporting the need to incorporate both AR and MA elements into the ARIMA model. The patterns observed in these plots assist in selecting the appropriate ARIMA (1,1,1) configuration. Subsequently, the presence of long memory was tested, and the results are revealed in Table 3 below.

Table 3: Results of the Test for Long Memory

Test	Statistic	Std. Error	t-Statistic	p-value
Hurst Exponent	1.130208	0.017851	35.302920	0.000000e+00
Rescaled Range (R/S)	975.288939	NaN	NaN	4.402908e-43
GPH Estimator	0.932781	1.220760	0.764099	4.448082e-01
Lo Modified R/S	5.277607	NaN	NaN	5.104633e-03
ARFIMA (d)	0.998766	0.001605	622.260815	0.000000e+00

The result of the long memory is in Table 4, and the investigation demonstrates remarkably high and statistically significant Hurst and ARFIMA of the d estimates (d≈0.999). This signifying an extremely persistent relationship concerning both mean and variance. This observation aligns in the same direction but is substantially more pronounced than the dual long memory reported in African and CEE stock markets, where the “d” values typically range from 0 to 0.4 (Benbachir, 2025; Kasman et al., 2009; Lahmiri & Bekiros, 2021). Such pronounced persistence evidently contradicts weak form of market efficiency and implies that disturbances in the Nigerian market yield effects that last for an extended period, as similarly noted in the NGX cited in Benbachir (2025) and the returns on Nigerian market index in Subair and Arewa (2020). The results of the ARCH LM Test are shown in Table 3 to determine the present of volatility.

Table 4: Results for ARCH LM Test

Lag	LM Statistic	LM p-value	F Statistic	F p-value
5	0.002939	0.000	0.000587	0.000
10	0.007548	0.000	0.000752	0.000
15	0.013061	0.000	0.000866	0.000

Table 3 illustrates the findings from the Autoregressive Conditional Heteroskedasticity (ARCH) Lagrange Multiplier (LM) test performed at lag intervals of 5, 10, and 15 to assess the existence of ARCH influences (which refers to fluctuations in volatility over time) in the dataset. The findings indicate that the LM statistics show a modest increase with longer lags (0.002939 at lag 5, 0.007548 at lag 10, and 0.013061 at lag 15), signifying a certain level of reliance in variance over time. Moreover, it is crucial to note that the LM p-values are uniformly 0.000, falling significantly below the standard significance threshold of 0.05. Additionally, the corresponding F-statistics and their p-values (0.000) further strengthen this observation. Given that all p-values are under 0.05, the null hypothesis asserting no ARCH effect is dismissed at every level. This indicates robust statistical backing for the presence of heteroskedasticity (volatility clustering) in the dataset. The outcomes of the ARCH LM test evidently show that the dataset demonstrates considerable conditional heteroskedasticity across every lag length evaluated, validating the appropriateness of volatility models like GARCH for subsequent analysis. Furthermore, the results of the FIGARCH model parameters were estimated and the results are shown in Table 5 below.

Table 5: FIGARCH Model Parameter Estimation

Parameter	Estimate	Std.Error	t-value	p-value
μ	937.007705	14.150360	66.217939	0.000000
AR (1)	1.000000	0.000249	4012.868397	0.000000
MA (1)	0.176716	0.014574	12.125363	0.000000
ω	38.093219	4.828464	7.889304	0.000000
α_1	0.000041	0.037928	0.001069	0.999147
β_1	0.718726	0.027508	26.127488	0.000000
D	1.000000	0.001154	866.796801	0.000000
v	2.508634	0.031506	79.623803	0.000000
Information criterion		Ljung-Box Test (p-value)	ARCH-LM Test (p-value)	
AIC	8.153416	0.9182	1.0000	
BIC	8.168843			
HQIC	8.153403			
SIC	8.158952			

The results of FIGARCH model parameter estimation is contain in Table 5. In the FIGARCH framework, the mean ($\mu \approx 937$) and AR (1) ≈ 1 are significantly substantial, indicating a near unit root process for the mean and a strong reliance on historical index values. Additionally, the MA (1) component is also significantly positive, reflecting immediate adjustments. Within the variance equation, ω and β_1 (≈ 0.72) are of high significance, denoting a positive baseline variance alongside strong persistence in volatility; the fractional parameter D=1.0 ($p<0.001$) indicates a remarkably slow, hyperbolic decay of volatility shocks. This aligns with FIGARCH applications that identify long-term volatility dependence in stock and commodity markets (Nazarian et al., 2013; Basira et al., 2024; Bollerslev & Mikkelsen, 1996). The results of ARFIMA-GARCH model parameter estimation are shown in Table 6 below.

Table 6: ARFIMA-GARCH Model Parameter Estimation

Parameter	Estimate	Std. Error	t-value	p-value
M	936.585111	14.120760	66.326819	0.000000
AR (1)	0.999858	0.000348	2873.436641	0.000000
MA (1)	0.122912	0.020556	5.979268	0.000000
d (ARFIMA)	0.059761	0.015661	3.815864	0.000000
Ω	37.217735	7.051900	5.277689	0.000000
α_1	0.275591	0.033863	8.138288	0.000000
β_1	0.723409	0.025700	28.148637	0.000000
V	2.509395	0.075311	33.320618	0.000000
Information criterion		Ljung-Box Test (p-value)	ARCH-LM Test (p-value)	
AIC	8.147807	0.9699	1.000	
BIC	8.163234			
HQIC	8.147794			
SIC	8.153343			

Table 6 shows the results of the ARFIMA-GARCH model parameter estimation. The ARFIMA-GARCH model presents a significant fractional differencing parameter in the mean, d≈0.06, reflecting a mild yet notable long memory in returns, while both α_1 and β_1 are significant, with their sum close to one, highlighting substantial volatility persistence. This trend is consistent with findings that ARFIMA-GARCH offers better fit and forecast accuracy compared to standard ARMA-GARCH models for exchange rates, stock indices, and macroeconomic variables (Adewole, 2024; Magaji & Garba, 2023; Contreras Reyes et al., 2024). The results of the ARFIMA-FIGARCH model estimation are shown in Table 7.

Table 7: ARFIMA-FIGARCH model estimation.

Parameter	Estimation			P-value				Nyblom Stability Test
μ	936.585111			0.000000				0.02064
D	0.0604			0.000000				0.000000
AR(1)	0.999858			0.000000				0.27706
MA(1)	0.122912			0.000000				0.15924
ARFIMA	0.059761			0.000136				0.24327
Ω	37.217735			0.000000				1.82639
A	0.275591			0.000000				2.37254
B	0.723409			0.000000				2.57615
D	1.000			0.000				0.000000
Skew	2.509395			0.000000				1.69034
Joint Nyblom Stability Statistic								11.4913
Information criterion	Akaike	Bayes	Shibata	Hannan-Quinn	Asymptotic Critical Values			
	8.1482	8.166	8.148	8.155		10%	5%	1%
					Joint Statistic:	1.89	2.11	2.59
					Individual Statistic	0.35	0.47	0.75

Table 7 shows the results of the ARFIMA-FIGARCH model estimation. The Estimates from the ARFIMA-FIGARCH model reveal that all main parameters in the mean (μ, AR (1), MA (1), ARFIMA d) and variance (Ω, α, β, D) achieve statistical significance at standard levels, affirming the presence of dual long memory in both returns and volatility. This observation is consistent with studies concerning prominent African and global markets where ARFIMA-FIGARCH effectively encapsulates dual long memory behavior and challenges weak form efficiency (Benbachir, 2025; Lahmiri & Bekiros, 2021; Jiang et al., 2023), yet the values obtained for d and D are at the higher end of the spectrum, indicating even more robust persistence than is typically reported. The results of the diagnostic tests for the ARFIMA-FIGARCH model are shown in Table 8 below.

Table 8 Diagnostic tests for the ARFIMA-FIGARCH model

Statistics	**P-value**	P-value
Weighted Ljung-Box Test on Standardized Residuals		
Lag[1]	0.410	0.5220
Lag[5]	1.854	0.9794
Lag[9]	2.313	0.9670
Weighted Ljung-Box Test on Standardized Squared Residuals		
Lag[1]	0.000661	0.9795
Lag[5]	0.001498	1.0000
Lag[9]	0.002545	1.0000
Weighted ARCH LM Test		
ARCH Lag[3]	0.0007369	0.9783
ARCH Lag[5]	0.0014923	1.0000
ARCH Lag[7]	0.0020871	1.0000
Sign Bias Test		
Sign Bias (t)	0.93524	0.3497
Negative Sign Bias (t)	0.10578	0.9158
Positive Sign Bias (t)	0.01099	0.9912
Joint Effect (t)	1.03936	0.7917
Adjusted Pearson Goodness-of-Fit Test		
Group 20	101.6	2.697e-13
Group 30	114.0	4.930e-12
Group 40	123.0	1.237e-10
Group 50	136.0	4.088e-10

Table 8 contains the results of the diagnostic tests for the ARFIMA-FIGARCH model. The Nyblom stability statistics predominantly remain beneath critical levels for most parameters, signifying a consistent stability of parameters over time. Coupled with elevated p values from the residual Ljung–Box and ARCH LM tests, alongside non-significant sign bias statistics, this indicates that the ARFIMA–FIGARCH model is effectively specified for the ASI: it adequately accounts for serial correlation, volatility clustering, and long memory while avoiding residual autocorrelation or unaddressed heteroscedasticity. Comparable diagnostic adequacy has been observed when applying ARFIMA–FIGARCH to indices from Africa and cryptocurrencies (Benbachir, 2025; Jiang et al., 2023; Lahmiri & Bekiros, 2021). The comparison of mean forecasts for the FIGARCH VS ARFIMA-GARCH VS ARFIMA-FIGARCH for the All-share index is shown in the plot below.

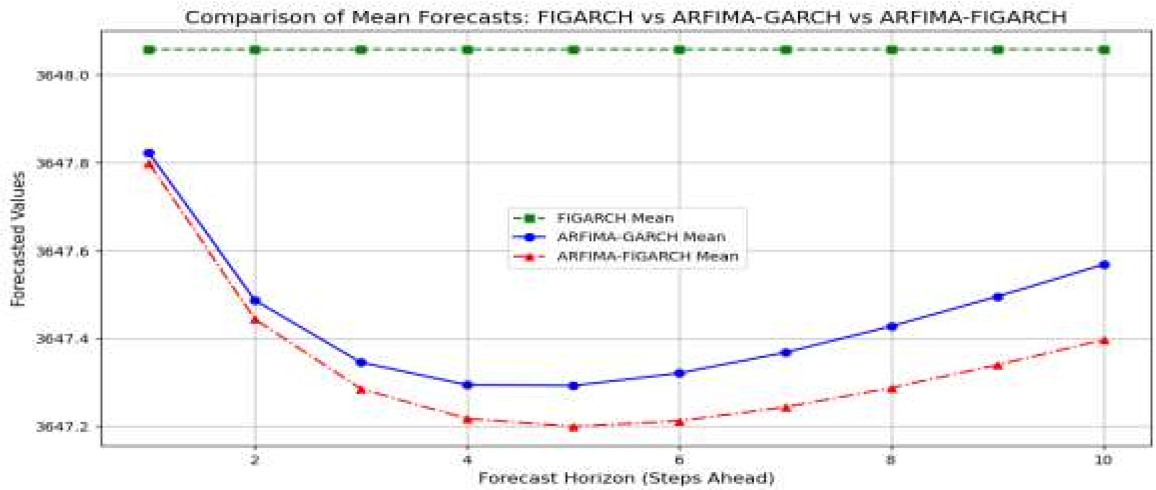

Figure 3: Comparison of Mean Forecasts: FIGARCH VS ARFIMA-GARCH VS ARFIMA-FIGARCH on ASI.

The illustration in Figure 3 demonstrates that the average projection from the FIGARCH model appears as a flat and level line, whereas both ARFIMA–GARCH and ARFIMA–FIGARCH models show a curvilinear trend. This suggests that the FIGARCH model posits a stable conditional mean over time, implying minimal or absent persistence in the average returns of the All Share Index (ASI). On the other hand, the curvature in the ARFIMA–GARCH and ARFIMA–FIGARCH indicates the existence of long-memory characteristics in the mean process, which captures slow changes and dependence patterns within returns. Consequently, this means that the models based on ARFIMA are more appropriate for analyzing and forecasting the ASI mean returns because they incorporate persistence and fractional integration, which are typical in financial time series. This conclusion is supported by earlier research conducted by Tuaneh, Deebom, and Akah (2025), highlighting that financial returns frequently display long memory and that ARFIMA-type models surpass short-memory models in effectively capturing these behaviors. In the context of the Nigerian stock market, it indicates that returns are not completely random; instead, they reveal a gradual dependence that can be used by investors and policymakers to improve forecasting and make strategic choices. Furthermore, the comparison of volatility forecasts for the FIGARCH VS ARFIMA-GARCH VS ARFIMA-FIGARCH for ASI were investigated and the results are shown in Figure 4 below.

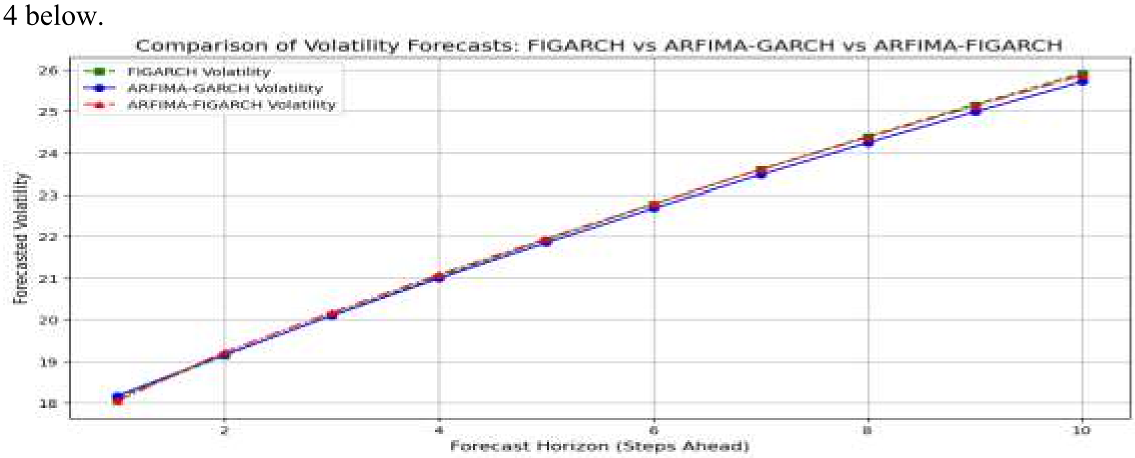

Figure 4: Comparison Of Volatility Forecasts: FIGARCH VS ARFIMA-GARCH VS ARFIMA-FIGARCH on ASI

The visualization in Figure 4 indicates that the forecasts for volatility (variance) from all three models—FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH—display a linear trend. This indicates that the models reach a stable long-term variance, reflecting consistency in the volatility predictions over the forecasting period. The implication is that while these models can capture long-memory effects in volatility, the forecast period examined might not reveal substantial variations, resulting in a smoothing effect. This aligns with observations made by Deebom, Essi, and Emeka (2021) who remarked that long-memory GARCH-type models typically generate persistent yet mean-reverting volatility forecasts. Regarding the Nigerian stock market, this infers that despite potential short-term volatility shocks, the market is inclined to return to stability in the long run, showing a certain level of resilience. Nevertheless, the absence of fluctuations in the predicted variance could also signify challenges in detecting abrupt structural changes or external shocks within the market. Also, multi-step forecasts for the mean and conditional volatility of FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH Models were investigated and the results are shown in Table 9 below.

Table 9: Multi-step forecasts of mean and conditional volatility of FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH Models

steps	FIGARCH (μ)	ARFIMA-GARCH_(μ)	ARFIMA-FIGARCH(μ)	FIGARCH Volatility	ARFIMA-GARCH Volatility	ARFIMA-FIGARCH Volatility
1	3648.057196	3647.820959	3647.797146	18.116608	18.168672	18.044330
2	3648.057223	3647.486545	3647.443516	19.139335	19.156939	19.220465
3	3648.057250	3647.345133	3647.284749	20.109920	20.095747	20.174810
4	3648.057277	3647.293651	3647.216746	21.035771	20.991735	21.080780
5	3648.057304	3647.292001	3647.199006	21.922555	21.850172	21.949210
6	3648.057331	3647.320385	3647.211533	22.774837	22.675324	22.784558
7	3648.057358	3647.367746	3647.243163	23.596354	23.470703	23.590344
8	3648.057385	3647.427416	3647.287160	24.390217	24.239240	24.369501
9	3648.057412	3647.495141	3647.339232	25.159043	24.983414	25.124506
10	3648.057440	3647.568105	3647.396537	25.905062	25.705340	25.857476

Table 9 shows the results of the multi-step forecasts of mean and conditional volatility of FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH Models. The multi-step forecasts illustrate that the average predictions from FIGARCH, ARFIMA–GARCH, and ARFIMA–FIGARCH are very similar and stable, whereas conditional volatility grows with the forecast horizon, indicating increasing uncertainty, as noted in long-term forecasting within Japanese, Asian, and commodity markets utilizing ARFIMA/FIGARCH models (Lux & Kaizoji, 2007; Duppati et al., 2017; Basira et al., 2024).

5.0 CONCLUSION

This research offers new empirical findings regarding extreme long-term memory persistence within the Nigerian stock market, showing fractional differencing parameters nearing one ($d \approx 0.999$), which significantly surpasses typical outcomes seen in both African and worldwide markets. This marks a key advancement compared to earlier research, where levels of persistence were notably weaker, positioning the Nigerian stock market as one that demonstrates hyper-persistent dynamics. Additionally, it reveals that both the returns and volatility associated with the All-Share Index display a dual long-memory characteristic, suggesting that disturbances in the market have extensive and slowly diminishing impacts. In contrast to standard studies where persistence is moderate, this near unity root characteristic implies that the Nigerian market strongly diverges from weak-form efficiency, demonstrating hyper-persistent dynamics. This indicates that previous data continues to hold considerable sway over upcoming market movements. The use and evaluation of FIGARCH, ARFIMA-GARCH, and ARFIMA-FIGARCH models enhance methodological contributions by showing that hybrid models that integrate dual long memory are more effective at capturing both return dependency and the clustering of volatility.

Moreover, while the FIGARCH model captures persistence in long-term volatility, the models based on ARFIMA (ARFIMA-GARCH and ARFIMA-FIGARCH) exhibit a greater capability in modeling the dynamic nature of average returns, as shown by their curved forecasting patterns. The ARFIMA-FIGARCH model stands out as the most effective framework, adeptly capturing both mean and volatility persistence while successfully passing essential diagnostic tests for stability. The stable volatility forecasts seen across the models indicate an enduring reliability, yet the escalating multi-step volatility forecasts reveal increasing uncertainty over prolonged timeframes.

The implications for the Nigerian stock market are significant. The occurrence of ultra-persistent shocks implies that market disruptions—stemming from changes in policy, economic instability, or external shocks—can lead to enduring ramifications, thereby heightening the risk exposure to investors. Concurrently, the observable dependence structure within returns opens avenues for predictability and strategic forecasting, which can be utilized by institutional investors and policymakers. The results emphasize the necessity for strong regulatory frameworks, enhanced market transparency, and sophisticated risk management strategies to improve market efficiency and stability in Nigeria. Also, The research adds value to theoretical understanding and practical applications by highlighting that the integration of dual long-memory models which is crucial for effectively modeling, predicting, and handling risks in developing markets such as Nigeria. The use of the combined ARFIMA–FIGARCH model marks an important advancement, as it effectively captures dual long-memory behavior while also exhibiting robust diagnostic adequacy and stability.

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