



# A Market Model of Quadratic Trend Functions Exhibiting Perturbation Changes.

<sup>1\*</sup>Onugu, C, <sup>2</sup>Obokare, O. A & <sup>3</sup>Amadi, I .U

<sup>1and 2</sup>Department of Mathematics/Statistics ,  
Federal University of Otuoke, Bayelsa State, Nigeria

\*Email: [onuguchristain.983@gmail.com](mailto:onuguchristain.983@gmail.com)

<sup>3</sup>Department of Mathematics/ Statistics Captain Elechi Amadi polytechnic, Rumuola Port  
Harcourt., email : [amadiuchennainno@gmail.com](mailto:amadiuchennainno@gmail.com)

## ABSTRACT

This project considered the application of matrix algebra in modeling market capitalization trend functions for investment plans. Firstly, mathematical formulations of the problem were developed: to established share price rate of change for quadratic trend function; which have significant impacts on share prices of Access Bank and Fidelity respectively. This project considered actual versus predicted prices, mean and volatility which can be used for investment decisions. A perturbation analysis was conducted on a model predicting share prices of Access Bank and Fidelity Bank. Results showed the original model was robust, with variations falling within expected limits indicating stability in predictions. We developed propositions, and proved them such that can be useful to investors to identify potential buying opportunities and decision making. All computations were done in Matlab to facilitate the understanding of mathematical concepts.

**Keywords:** Share price, Banks, Matrices, Perturbation, Trend Functions

## 1.1 INTRODUCTION

In financial markets, understanding and predicting share price movements is crucial for investors and analysts. A market model using quadratic trend functions with perturbation changes offers a way to capture complex behaviors of share prices, like non-linear trends and volatility. Quadratic functions can model curved trends in share prices, reflecting changes in market sentiment or economic factors. This is particularly relevant for banks like Access Bank and Fidelity Bank in Nigeria. The analysis of share price changes is a complex and multifaceted field that has garnered significant attention from investors, researchers and financial analysts share prices are influenced by a vast array of factors, including macroeconomic, indicators, company performance, market, sentiment and global events. More so, perturbation analysis examines how small changes (like in parameters or external factors) affect model predictions. This tests model robustness, if small changes cause big prediction swings, the model might be less reliable. Hence, quadratic trends and perturbations can reflect how banks share prices react to news or policy changes. Its predictions help investors anticipate movements for Access and Fidelity Banks based on trends and volatility. Understanding volatility helps investors manage risk by identifying potential price movements and adjusting their portfolios accordingly. Volatility contributes to market efficiency, as it reflects changes in market sentiment and incorporates new information into share prices. Also mean-reversion and long-term mean might revert to a long-term mean, reflecting equilibrium. The implication is that it guilds understanding of normal price levels for Access/Fidelity shares. Applying rate of change on the shares helps to reflects market momentum; rapid changes can signal volatility. This model helps investors assess risk, predict movements, and make informed decisions for banks like Access and Fidelity in Nigeria's market. On the other hand, matrix algebra provides a framework for modeling

the relationships between multiple assets, allowing researchers to analyze the covariance and correlations between stocks, bond and other financial instruments. By applying matrix algebra, researchers can identify potential arbitrage opportunities, optimize portfolio weights and develop more effective risk management strategies. According to [1] the topic of matrix has a lot of great content for discussion and research. It offers countless beautiful theorems that are straight forward and yet striking on their formulation, uncomplicated and yet ingenious in their proof, and diverge as well as powerful in their application.

Nevertheless, lots of authors has written extensively on stock prices,[2] examined a matrix application to stock market prices where an illuminating example is provided in diverse methods. In the work of [3] studied stochastic system with changes to measure the value of wealth for each corporate investor through linear and quadratic returns. Also, [4] Explored system of stochastic differential equations with importance on differences of drift parameter for stock market. In the same vain, [5] considered the stochastic analysis of two asset values. Due to the unstable nature of stock prices [6] looked at the stability and controllability for stock market prices with control. [7] investigated the applications of various stochastic volatility models in determining optimal investment strategies in the stock market. So far, [8] studied stochastic model of the fluctuation of stock market price. Many authors have extensively addressed the issues of stock prices such as [9 – 17].

In all, we were motivated due to the understanding of model stability which helps investors trust predictions, make informed decisions, and assess market behavior for these banks.

More so, this project is geared to develop empirical approach of studying matrix algebra and its applications for capital market plans. It is obvious that investors are indeed affected in their major decisions due to expected returns and to address the problem of price changes for modeling share prices. This encouraged us to come up with vital and good approaches that can stand to make informed decisions. Our novel idea compliments the works of [8] and widens the applicability of problem in this area of mathematical finance.

The arrangement of this paper is set as follows: Section 2.1 is Material and Methods, Section 3.1 presents results and discussion, while the paper is concluded in 4.1.

## 2.1 MATERIAL AND METHODS

In this Section, we review relevant basic foundations that would help in achieving the derivation.

**Definition 2.1:** In the context of share prices, a function can be thought of as a mathematical representation of the relationship between the share price of a bank and time. Example: Let’s say we want to model the share price of Access Bank as a function of time ( $t$ ) . We can represent its as:

$$K(t) = f(t) \tag{1.1}$$

where  $K(t)$  is the share price of Access Bank at time  $t$ , and  $f(t)$  is the function that describes the relationship between the share price and time.

**Definition 2.2:** A quadratic function is a polynomial function of degree two which means the highest power of the variable (usually  $x$  or  $t$ ) is two. Example : Let’s say we want to model the share price of Fidelity Bank using a quadratic function:

$$K(t) = at^2 + bt + c \tag{1.2}$$

where  $K(t)$  is the share price of Fidelity Bank at time, and  $a, b,$  and  $c$  are constants

**Definition 2.3:** A matrix is defined as a system of number arranged in a rectangular array of rows and columns enclosed in parenthesis.

**Definition 2.4:** Row and columns, the horizontal lines are called rows while the vertical lines are called columns.

**Definition 2.5:** When a matrix has  $m$  rows and  $n$  columns is known as  $m \times n$  or  $m$  by  $n$ . The pair  $(m, n)$  is called the dimension (order) which is denoted by

$$B = \begin{pmatrix} b_{11} & b_{12} & \cdots & \dots & | \\ b_{21} & b_{22} & \cdots & \dots & | \\ \vdots & \vdots & \cdots & \vdots & | \\ b_{m1} & b_{m2} & b_{m3} & b_{mn} & | \end{pmatrix} \quad (1.3)$$

Thus, matrices are denoted by capital letters and its entries (elements) by small letters.

This is a matrix  $B$  having one row and  $n$  columns.

$$B = (b_{11}, b_{12}, \cdots, \dots, b_{1n})_{1 \times n} \quad (1.4)$$

This is a matrix  $B$  that has one column and  $m$  rows.

$$B = \begin{pmatrix} b_{11} \\ b_{21} \\ \vdots \\ b_{m1} \end{pmatrix} = (b_{..})_{m \times 1} \quad (1.5)$$

This is  $m \times 1$ .

**Definition 2.6:** This is a matrix having the same number of rows and columns. That means  $m = n$ . The square matrix defined thus:

$$B = \begin{pmatrix} b_{11} & b_{12} & \cdots & \dots & | \\ b_{21} & b_{22} & \cdots & \dots & | \\ \vdots & \vdots & \cdots & \vdots & | \\ b_{m1} & b_{m2} & b_{m3} & b_{mn} & | \end{pmatrix} = (b_{..})_{n \times n} \quad (1.6)$$

**Definition 2.7:** Derivative of  $Kx^n$ . If  $y = Kx^n$

where  $K$  is a constant and  $n$  is also a constant, then  $\frac{dy}{dx} = Knx^{n-1}$ .

**2.2 The Heston Model:** Heston proposed a stochastic volatility model where the underlying asset behavior was characterized by the following risk-neutral dynamics:

$$\left. \begin{aligned} dS_t &= rS_t dt + \sqrt{V_t} S_t dW_t^1 \\ dX_t &= \alpha(\bar{X} - X_t) dt + \eta \sqrt{X_t} dW_t^2 \\ dW_t^1 dW_t^2 &= \rho dt \end{aligned} \right\} \quad (1.7)$$

where  $S_t$  is the price of the underlying asset at time  $t$ ,  $r$  is the risk free rate,  $V_t$  is the variance at time  $t$ ,  $\bar{X}$  is the long-term variance,  $\alpha$  is the variance mean-reversion speed,  $\eta$  is the volatility of the variance process and  $dW_t^1 dW_t^2$  are two correlated Weiner processes, with correlation coefficient  $\rho$ .

### 2.3 Problem Formulation

We employ matrix algebra to model the share prices of Access Bank, Fidelity Bank. Considering Quadratic trend. This formulation enables us to capture the complex dynamics of the investment process, incorporating the trend exhibited by the share prices of these banks.

Let  $A_i(t)$ , and  $F_i(t)$  represents the share prices of these banks at time  $t$ , where  $i = 1, 2, 3$  corresponding to the respective trends. Specifically:  $A_1(t)$ , and  $F_1(t)$  follows a quadratic trend, Suppose the future investments plans, having a good knowledge and understanding of a fair market and according to historical data where share prices are represented in  $3 \times 3$  matrices; connecting (1.7) the investment dynamical systems is shown as follows:

➤ **Matrix of Quadratic trend function :**

$$A_t(t) = \begin{pmatrix} (\theta_0 - K_x) X'_{011} t^2 & (\theta_0 - K_x) X'_{012} t^2 & (\theta_0 - K_x) X'_{013} t^2 \\ (\theta_0 - K_x) X'_{021} t^2 & (\theta_0 - K_x) X'_{022} t^2 & (\theta_0 - K_x) X'_{023} t^2 \\ (\theta_0 - K_x) X'_{031} t^2 & (\theta_0 - K_x) X'_{032} t^2 & (\theta_0 - K_x) X'_{033} t^2 \end{pmatrix} \quad (1.8)$$

$$F_t(t) = \begin{pmatrix} (\theta_0 - K_x) X''_{011} t^2 & (\theta_0 - K_x) X''_{012} t^2 & (\theta_0 - K_x) X''_{013} t^2 \\ (\theta_0 - K_x) X''_{021} t^2 & (\theta_0 - K_x) X''_{022} t^2 & (\theta_0 - K_x) X''_{023} t^2 \\ (\theta_0 - K_x) X''_{031} t^2 & (\theta_0 - K_x) X''_{032} t^2 & (\theta_0 - K_x) X''_{033} t^2 \end{pmatrix} \quad (1.9)$$

where  $A_t(t)$ , and  $F_t(t)$  are the share prices of Access Bank, and Fidelity Bank at time  $t$  under quadratic trend functions,  $X'_{011} + \dots$  and  $X''_{011} + \dots$  represents monthly initial share prices of the two banks,  $\theta_0$  and  $K_x$  represents the long term mean of the share prices and the speed of mean-reversion parameter respectively.

### 2.3.1 Method of Analysis

Here, we apply the method of rate of change in solving (1.8-1.9) according to their respective trend functions. The rate of change of an investment refers to how the value of the investment changes over time.

$$\frac{dA_t(t)}{dt} = \begin{pmatrix} 2(\theta_0 - K_x)X'_{011}t & 2(\theta_0 - K_x)X'_{012}t & 2(\theta_0 - K_x)X'_{013}t \\ 2(\theta_0 - K_x)X'_{021}t & 2(\theta_0 - K_x)X'_{022}t & 2(\theta_0 - K_x)X'_{023}t \\ 2(\theta_0 - K_x)X'_{031}t & 2(\theta_0 - K_x)X'_{032}t & 2(\theta_0 - K_x)X'_{033}t \end{pmatrix} \quad (1.10)$$

$$\frac{dF_t(t)}{dt} = \begin{pmatrix} 2(\theta_0 - K_x)X''_{011}t & 2(\theta_0 - K_x)X''_{012}t & 2(\theta_0 - K_x)X''_{013}t \\ 2(\theta_0 - K_x)X''_{021}t & 2(\theta_0 - K_x)X''_{022}t & 2(\theta_0 - K_x)X''_{023}t \\ 2(\theta_0 - K_x)X''_{031}t & 2(\theta_0 - K_x)X''_{032}t & 2(\theta_0 - K_x)X''_{033}t \end{pmatrix} \quad (1.11)$$

### 2.3.2 Actual and Predicted Share prices

Here, in order to ascertain the robustness of the investment mathematical models we compare the actual and predicted share. Hence, let each of the share prices and their predicted prices of the banks be represented as follows:

$(A_1, A_2, A_3, \dots)$  and  $(F_1, F_2, F_3, \dots)$  for Quadratic trend function during the trading.

### 2.4 Perturbations on Investment of Shares

Here, by introducing perturbations into investment of shares to identify potential weakness or vulnerability. Suppose we now perturb equations in (1.3-1.4) by adding a small random amount to each equation which gives as follows:

$$\frac{d'A_t(t)}{dt} = \begin{pmatrix} 2(\theta_0 - K_x)X'_{011}t & 2(\theta_0 - K_x)X'_{012}t & 2(\theta_0 - K_x)X'_{013}t \\ 2(\theta_0 - K_x)X'_{021}t & 2(\theta_0 - K_x)X'_{022}t & 2(\theta_0 - K_x)X'_{023}t \\ 2(\theta_0 - K_x)X'_{031}t & 2(\theta_0 - K_x)X'_{032}t & 2(\theta_0 - K_x)X'_{033}t \end{pmatrix} + \lambda \quad (1.12)$$

$$\frac{d'F_t(t)}{dt} = \begin{pmatrix} 2(\theta_0 - K_x)X''_{011}t & 2(\theta_0 - K_x)X''_{012}t & 2(\theta_0 - K_x)X''_{013}t \\ 2(\theta_0 - K_x)X''_{021}t & 2(\theta_0 - K_x)X''_{022}t & 2(\theta_0 - K_x)X''_{023}t \\ 2(\theta_0 - K_x)X''_{031}t & 2(\theta_0 - K_x)X''_{032}t & 2(\theta_0 - K_x)X''_{033}t \end{pmatrix} + \lambda \quad (1.13)$$

where  $\lambda$  is the random perturbation.

### 2.5. Analysis of Share Price Due to Approximate Changes

Convincingly rate of change in share prices of the three banks connotes the value of shares. Following the method of Amadi et al. (2025), in financial market, the value of stocks grows considerably according to different dimensions over time. The curves of growth may vary due to some random disturbances. The shapes of this curve will be realistic if integration is applied for its appropriate approximations as this will provide management, investors, policy makers with valuable insight about the performance of the banks under study and its standing in the market, hence we state and prove the following propositions:

**Proposition 2.1:** In view of an aspect of random variability of a stock, [8]. Let  $A(t)$  represents the aggregate intrinsic value of share prices. Also let  $\theta_0^9$  represents changes on interest rate or volatility with respect to the trading seasonal periods, which is the value of an investment over time and  $Sin\theta_0$  is the nature and capacity of asset both at time,  $t$ . Suppose the dividend is declared at time,  $t$ , the shape of the curve grows to a process  $A(t) = \lambda^2 \theta_0^9 Sin\theta_0$  where  $9$  is the number of trading days,  $\lambda$  is a constant to be determined; all on the bounded interval  $[0, 2\pi]$ . Hence the dynamics governing the entire process is:

$$A(t) = \int_0^{2\pi} (\lambda^2 \theta_0^9 Sin\theta_0) d\theta_0$$

**Proof**

We want to show how share price of the three banks grow significantly according to certain price index in different periods.

Using Nedu's method of integration by parts

$$\begin{aligned} \int_0^{2\pi} (\lambda^2 \theta_0^9 Sin\theta_0) d\theta_0 &= \lambda^2 \left[ \begin{aligned} &\theta^9 \int Sin\theta_0 d\theta_0 - 9\theta_0^8 \int Sin\theta_0 d\theta_0 + 72\theta_0^7 \int Sin\theta_0 d\theta_0 - 504\theta_0^6 \int Sin\theta_0 d\theta_0 + \\ &3024\theta_0^5 \int Sin\theta_0 d\theta_0 - 15120\theta_0^4 \int Sin\theta_0 d\theta_0 + 60480\theta_0^3 \int Sin\theta_0 d\theta_0 \\ &- 181440\theta_0^2 \int Sin\theta_0 d\theta_0 + 362880\theta_0 \int Sin\theta_0 d\theta_0 - 362880 \int Sin\theta_0 d\theta_0. \end{aligned} \right] \\ &= \lambda^2 \left[ \begin{aligned} &\theta_0^9 (-Cos\theta_0) - 9\theta_0^8 (-Sin\theta_0) + 72\theta_0^7 (Cos\theta_0) - 504\theta_0^6 (Sin\theta_0) + 3024\theta_0^5 (-Cos\theta_0) \\ &+ 15120\theta_0^4 (-Sin\theta_0) + 60480\theta_0^3 (-Cos\theta_0) - 181440\theta_0^2 (Sin\theta_0) + 362880\theta_0 (-Cos\theta_0) + 362880 (Sin\theta_0) \end{aligned} \right] \\ &= \lambda^2 \left[ \begin{aligned} &-\theta_0^9 Cos\theta_0 + 9\theta_0^8 Sin\theta_0 + 72\theta_0^7 Cos\theta_0 - 504\theta_0^6 Sin\theta_0 - 3024\theta_0^5 Cos\theta_0 + 15120\theta_0^4 Cos\theta_0 \\ &- 60480\theta_0^3 Cos\theta_0 - 181440\theta_0^2 Sin\theta_0 - 362880\theta_0 Cos\theta_0 + 362880 Sin\theta_0. \end{aligned} \right] \end{aligned}$$

Taking the upper and lower boundaries gives the following

$$= -\lambda^2 \left[ \begin{aligned} &512\pi^9 Cos2\pi + 2304\pi^8 Sin2\pi + 92160\pi^7 Cos2\pi - 32256\pi^6 Sin2\pi - 96768\pi^5 Cos2\pi + 241920\pi^4 Sin2\pi \\ &- 4838400\pi^3 Cos2\pi - 72560\pi^2 Sin2\pi - 725760\pi Cos2\pi + 362880 Sin2\pi. \end{aligned} \right]$$

**Proposition 2.2:** In considering, periodic fluctuations in the investment returns which is a seasonal cycle such as sales of shares that peak in the spring and trough in the fall. Let  $B(t)$  represent intrinsic value of the shares. Then, let  $\phi_t$  be the unit price of the shares,  $Cos\phi_t$  is the capacity and nature of the asset all at time  $t$ . Supposing the shape of the curve grows to a process  $B(t) = \lambda^2 \phi_t^9 Cos\phi_t$ ,  $\lambda$  is a constant to be determined; all on the bounded interval  $[0, 2\pi]$  and hence the future value dynamics follows the process:

$$B(t) = \int_0^{2\pi} (\lambda^2 \phi_T^9 \text{Cos} \phi_T) d\phi_T$$

**Proof**

We want to show how share price of the three banks grow significantly according to certain price index in different periods.

Using Nedu’s method of integration by parts.

$$\begin{aligned} \int_0^{2\pi} (\lambda^2 \phi_T^9 \text{Cos} \phi_T) d\phi_T &= \lambda^2 \left[ \begin{aligned} &\phi_T^9 \int \text{Cos} \phi_T d\phi_T - 9\phi_T^8 \int \text{Cos} \phi_T d\phi_T + 72\phi_T^7 \int \text{Cos} \phi_T d\phi_T - 504\phi_T^6 \int \text{Cos} \phi_T d\phi_T + \\ &3024\phi_T^5 \int \text{Cos} \phi_T d\phi_T - 15120\phi_T^4 \int \text{Cos} \phi_T d\phi_T + 60480\phi_T^3 \int \text{Cos} \phi_T d\phi_T \\ &- 181440\phi_T^2 \int \text{Cos} \phi_T d\phi_T + 362880\phi_T \int \text{Cos} \phi_T d\phi_T - 362880 \int \text{Cos} \phi_T d\phi_T. \end{aligned} \right] \\ &= \lambda^2 \left[ \begin{aligned} &\phi_T^9 (\text{Sin} \phi_T) - 9\phi_T^8 (-\text{Cos} \phi_T) + 72\phi_T^7 (-\text{Sin} \phi_T) - 504\phi_T^6 (\text{Cos} \phi_T) + 3024\phi_T^5 (\text{Sin} \phi_T) - 15120\phi_T^4 (-\text{Cos} \phi_T) \\ &+ 60480\phi_T^3 (-\text{Sin} \phi_T) - 181440\phi_T^2 (\text{Cos} \phi_T) + 362880\phi_T (\text{Sin} \phi_T) - 362880(-\text{Cos} \phi_T). \end{aligned} \right] \\ &= \lambda^2 \left[ \begin{aligned} &\phi_T^9 \text{Sin} \phi_T + 9\phi_T^8 \text{Cos} \phi_T - 72\phi_T^7 \text{Sin} \phi_T - 504\phi_T^6 \text{Cos} \phi_T + 3024\phi_T^5 \text{Sin} \phi_T + 15120\phi_T^4 \text{Sin} \phi_T \\ &- 60480\phi_T^3 \text{Sin} \phi_T - 181440\phi_T^2 \text{Cos} \phi_T + 362880\phi_T \text{Sin} \phi_T + 362880 \text{Cos} \phi_T. \end{aligned} \right] \end{aligned}$$

Taking the upper and lower boundaries gives as follows

$$= \lambda^2 \left[ \begin{aligned} &512\pi^9 \text{Sin} 2\pi + 2304\pi^8 \text{Cos} 2\pi - 9216\pi^7 \text{Sin} 2\pi - 32256\pi^6 \text{Cos} 2\pi + 96768\pi^5 \text{Sin} 2\pi + 241920\pi^4 \text{Sin} \phi_T \\ &- 483840\pi^3 \text{Sin} 2\pi - 725760\pi^2 \text{Cos} 2\pi + 725760\pi \text{Sin} 2\pi + 3628880 \text{Cos} 2\pi. \end{aligned} \right]$$

**Proposition 2.3:** Assuming the investment of shares grows exponentially considering some factors such as sales that increases both in winter or summer whose unit price of the stock, capacity and nature of the asset is  $e^{2\alpha}$  and  $(\alpha^9 + \beta\alpha) d\alpha$  respectively all at time  $t$ . The entire function becomes  $C(t)$ . Suppose the sales of shares increases to a curve of a fundamental processes known as  $C(t) = \lambda^2 e^{2\alpha} (\alpha^9 + \beta\alpha) d\alpha$ ,  $\lambda$  and  $\beta$  is a constant to be determined; all on the bounded interval  $[0, \pi]$  therefore the process of the investment is defined as:

$$C(t) = \int_0^{\pi} \lambda^2 e^{2\alpha} (\alpha^9 + \beta\alpha) d\alpha.$$

**Proof**

We want to show how share price of the three banks grow significantly according to certain price index with exponential over time.

Using Nedu’s method of integration by parts

$$\int_0^2 \lambda^2 e^{2\alpha} (\alpha^9 + \beta\alpha) d\alpha = \lambda^2 \left[ \begin{aligned} & (\alpha^9 + \beta\alpha) \int e^{2\alpha} d\alpha - (9\alpha^8 + \beta) \int e^{2\alpha} d\alpha + (72\alpha^7) \int e^{2\alpha} d\alpha - 504\alpha^6 \int e^{2\alpha} d\alpha \\ & + 3024\alpha^5 \int e^{2\alpha} d\alpha - 15120\alpha^4 \int e^{2\alpha} d\alpha + 60480\alpha^3 \int e^{2\alpha} d\alpha - 181440\alpha^2 \int e^{2\alpha} d\alpha \\ & + 362880\alpha \int e^{2\alpha} d\alpha - 362880 \int e^{2\alpha} d\alpha. \end{aligned} \right]$$

$$= \lambda^2 \left[ \begin{aligned} & (\alpha^9 + \beta\alpha) \left( \frac{e^{2\alpha}}{2} \right) - (9\alpha^8 + \beta) \left( \frac{e^{2\alpha}}{4} \right) + (9\alpha^7) (e^{2\alpha}) - \left( \frac{63}{2} \alpha^6 \right) (e^{2\alpha}) + \left( \frac{189}{2} \alpha^5 \right) (e^{2\alpha}) - \left( \frac{945}{4} \alpha^4 \right) (e^{2\alpha}) \\ & + \left( \frac{945}{2} \alpha^3 \right) (e^{2\alpha}) - \left( \frac{2835}{4} \alpha^2 \right) (e^{2\alpha}) + (720\alpha) (e^{2\alpha}) - (360) (e^{2\alpha}). \end{aligned} \right]$$

$$= \lambda^2 \left[ \begin{aligned} & (\alpha^9 + \beta\alpha) \left( \frac{e^{2\alpha}}{2} \right) - (9\alpha^8 + \beta) \left( \frac{e^{2\alpha}}{4} \right) + (72\alpha^7) \left( \frac{e^{2\alpha}}{8} \right) - (504\alpha^6) \left( \frac{e^{2\alpha}}{16} \right) + (3024\alpha^5) \left( \frac{e^{2\alpha}}{32} \right) - (15120\alpha^4) \\ & \left( \frac{e^{2\alpha}}{64} \right) + (60480\alpha^3) \left( \frac{e^{2\alpha}}{128} \right) - (181440\alpha^2) \left( \frac{e^{2\alpha}}{256} \right) + (362880\alpha) \left( \frac{e^{2\alpha}}{504} \right) - (362880\alpha^7) \left( \frac{e^{2\alpha}}{1008} \right). \end{aligned} \right]$$

Taking the upper and lower boundaries gives the following

$$= \lambda^2 \left[ \begin{aligned} & (512 + 2\beta) \left( \frac{e^4}{2} \right) - (2304 + \beta) \left( \frac{e^4}{4} \right) + (1152) (e^4) - \left( \frac{4032e^4}{2} \right) + \left( \frac{6048e^4}{2} \right) - \left( \frac{15120e^4}{4} \right) + \\ & \left( \frac{7560e^4}{2} \right) - \left( \frac{11340e^4}{4} \right) + (1440e^4) - (360e^4) - \left( \frac{\beta}{4} \right) - 1 + 360. \end{aligned} \right]$$

### 3.1 Results and Discussion

This Section presents analyzed results whose methods are stated in Chapter three. Hence, we have the following parameter values:  $\theta_0 = 0.998$ ,  $K_x = 0.499$ , and  $t = 1$ . which were implemented using Matlab programming software:

➤ **The rate of change when the share prices is quadratic trend function at  $t = 1$  :**

$$\frac{dA_t(t)}{dt} = \begin{pmatrix} 401 & 71 & 118 \\ 70 & 90 & 84 \\ 119 & 88 & 369 \end{pmatrix}, \frac{dF_t(t)}{dt} = \begin{pmatrix} 406 & 52 & 139 \\ 51 & 113 & 72 \\ 131 & 71 & 375 \end{pmatrix}$$

The instability in predicted share prices for Access Bank and Fidelity Bank could mean: the prediction model might be struggling to capture complex market patterns. External factors like economic changes or investor sentiment are impacting prices unpredictably. This trend function captures non-linear relationship between variables. The results suggest that the quadratic trend has a significant impact on share prices, indicating that the rate of change in share prices is influenced by the quadratic term.

**Table 3.1: Comparison of Actual , Predicted prices and Measures of Stock Variables**

<b>Access Actual Share prices</b>	<b>Predicted Prices</b>	<b>Fidelity Actual Share prices</b>	<b>Predicted Prices</b>	<b>Mean prices of both banks</b>	<b>Volatility of both banks</b>
402	401	407	406	404	2.9439
71	71	52	52	61.5000	10.9697
118	118	139	139	128.5000	12.1244
70	70	51	51	60.5000	10.9697
90	90	113	113	101.5000	13.2791
84	84	72	72	78	6.9282
119	119	131	131	125	6.9282
88	88	71	71	79.5000	9.8150
370	369	376	375	372.5000	3.5119

Table 3.1, when actual and predicted share prices are closely aligned, it means your model’s doing a solid job: This is likely capturing the underlying patterns in the data pretty well. The market might be efficient making, making it harder to find undervalued or overvalued stocks. Also, if predictions match actual prices, there aren’t obvious opportunities to profit from price discrepancies. The mean of the share prices would give an average value, showing kinda like a middle ground between the two prices, see column 5 of Table 3.1. We talk about volatility in the context of share prices, we are referring to how much the price of a stock fluctuates over time. For Access Bank and Fidelity Bank, having volatility means their stock prices are changing either going up or down, see column 6.

**Table 3.2: The Perturbed Access Bank Matrix Model**

<b>Access Predicted Prices</b>	<b>Perturbed <math>\lambda = 0.01</math></b>	<b>Perturbed <math>\lambda = 0.02</math></b>	<b>Perturbed <math>\lambda = 0.03</math></b>	<b>Perturbed <math>\lambda = 0.04</math></b>	<b>Perturbed <math>\lambda = 0.05</math></b>
401	401.01	401.02	401.03	401.04	401.05
71	71.01	71.02	71.03	71.04	71.05
118	118.01	118.02	118.03	118.04	118.05
70	70.01	70.02	70.03	70.04	70.05
90	90.01	90.02	90.03	90.04	90.05
84	84.01	84.02	84.03	84.04	84.05
119	119.01	119.02	119.03	119.04	119.05
88	88.01	88.02	88.03	88.04	88.05
369	369.01	369.02	369.03	369.04	369.05

Table 3.3: The Perturbed Fidelity Bank Matrix Model

Fidelity Predicted Prices	Perturbed $\lambda = 0.01$	Perturbed $\lambda = 0.02$	Perturbed $\lambda = 0.03$	Perturbed $\lambda = 0.04$	Perturbed $\lambda = 0.05$
406	406.01	406.02	406.03	406.04	406.05
52	52.01	52.02	52.03	52.04	52.05
139	139.01	139.02	139.03	139.04	139.05
51	51.01	51.02	51.03	51.04	51.05
113	113.01	113.02	113.03	113.04	113.05
72	72.01	72.02	72.03	72.04	72.05
131	131.01	131.02	131.03	131.04	131.05
71	71.01	71.02	71.03	71.04	71.05
375	375.01	375.02	375.03	375.04	375.05

In Tables 3.2 and 3.3 respectively, the perturbation analysis showing robustness means the original model’s results are kinda holding up despite the tweaks. It implies model been stable, results are reliable within those limits and systems not super sensitive to small changes.

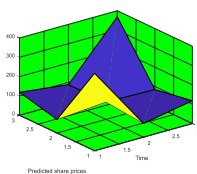


Figure 3.1: Surface view of Access Bank predicted Prices

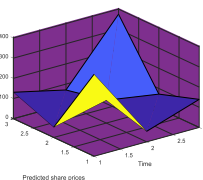


Figure 3.2: Surface view of Fidelity Bank predicted Prices

Figures 3.1 and 3.2 are descriptive illustrations of surface view results of Fidelity, Access and banks share prices. It describes levels of changes of existence on the value of share prices. The shapes of the surface view shows unsatisfactory amplitudes show-casing its uncertainty due to stochastic formations. The trajectories seen on the figures is the price history of stock market which is been determined by volatility.

#### 4.1 CONCLUSION

The perturbation analysis revealed the model’s predictions are reliable and stable under small changes. The alignment of actual and predicted prices suggests the model captures market patterns effectively. Mean share prices and volatility were considered in evaluating model performance. Stating and proving propositions were highly explored to make vital decisions for the management of the two Banks. In next paper, we shall be looking at testing with larger perturbations to check model limits, explore different scenarios for broader understanding and analyze volatility patterns to improve predictions.

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