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A Black-Scholes Model on Stock Market Prices: Comparative of Call Option Price Distributions

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ABSTRACT

This study presents an in-depth analysis of call option prices for two initial stock prices (40 and 50) under various volatility scenarios. The investigation aims to identify the underlying distribution of call option prices and examine the impact of strike prices on option values. The results show that the Weibull distribution provides the best fit for the call option prices, indicating a non-normal and skewed distribution. The study also examines the effect of strike price on call option values, highlighting the importance of strike price in determining option leverage and risk exposure. Specifically, the analysis reveals that in-the-money options provide higher leverage, while out-of-the-money options provide lower leverage. We developed proposition and proved it such that it can be useful to investors to identify potential buying opportunities and decision making. The findings of this study have significant implications for option pricing, risk management, and trading strategies, emphasizing the need for accurate modeling of call option prices and careful consideration of strike price selection.

Keywords: Call Option, Stock, Prices, Distributions, Strike prices

1.1 INTRODUCTION

The pricing of call options is a complex and challenging task that involves understanding the underlying stock price dynamics and the factors that affect option values. Call options are financial derivatives that give the holder the right, but not the obligation to buy an underlying asset at a predetermined price (strike price) on or before a certain date (expiration date). The Black-Scholes model, a widely used option pricing model, assumes a lognormal distribution for stock prices and provides a closed solution for European call options,[1]. However, empirical evidence suggests that stock prices often exhibit non-normal and skewed distributions, which can lead to inaccurate option pricing and risk management.

On the contrary, many researchers have applied Black-Scholes model in different ways; for instance, [2] considered the impact of Finite Difference approach in Valuating of Options. In similar manner, [3] stipulated that the rate of option lies on the underlying asset, which is frequently a stock, commodity, currency or an index. Hence [4] established a new technique of assessing pricing effects on the premise to reduce pricing bias. [5] applied the tempered fractional derivative to price a European-double-knock-out barrier option. [6] analyzed Black-Scholes model analysis and discovered that volatility is not constant in the real trading of options.

Though, [7] established that the Black-Scholes model has been a major advance in finance over a period of time. In the work of [8] posited that the Black–Scholes Option pricing model has long been in use for valuation of equity options. [9] proposed a high accurate method based on non-standard

Runge–Kutta method. In another dimension [10] considered the Laguerre neural network as a novel numerical algorithm with three layers of neurons for solving Black-Scholes (BS) equations. More recently [13] studied the perception of European Call option , the explicit price on the variations of maturity days is found. The use of Black-Scholes is enormous that is, why so many authors has extensively addressed on trading of option pricing such [11-12], and [15-19].

It is obvious that [13] has considered analysis of Black-Scholes model of option pricing with time varying parameters on assumption that two call option prices do not come from a common distribution through Komogorov Sminorf. The motivation for this study stems from the need to better understand the behavior of call option prices and factors that affect their values. The strike price is a critical component of option pricing, and its impact on call option values is not well understood. This study aims to fill this gap by analyzing the behavior of call option prices for two initial stock prices (40 and 50) under various volatility scenarios and examining the effect of strike price on call option values. The study uses statistical tests (Kolmogorov-Smirnov and Anderson-Darling) to determine the best-fitting distribution for the call option prices and analyzes the effect of strike price on call option values. The results provide insights into option pricing, risk management, and trading strategies. Our novel idea compliments previous efforts and extends the frontier of knowledge in this dynamic area of mathematics of finance.

The plan of this paper is set as follows: Section 2.1 is Mathematical framework, Section 3.1 Results and Discussion and conclusion is seen in Section 4.1.

2.1 Mathematical Framework

We present definitions capturing this dynamic area of mathematics of finance, hence we have the following:

Definition 2.1: Probability space: This is a triple $(\Omega, \mathrm{F}, \tilde{A})$ where Ω represents a set of sample space, F represents a collection of subsets of Ω, while $\wp$ is the probability measure defined on each event $\mathrm{A} \in \mathrm{F}$. The collection F is a σ -algebra or σ -field such as $\Omega \in \mathrm{F}$ and F is closed under the arbitrary unions and finite intersections. Hence it is called probability measure when the following condition holds.

(i) $$P(A) \geq 0 \ for\ all\ A \subset \Omega$$

(ii) $$P(\Omega) = 1$$

(iii) $$A, B \subset \Omega,\ A \cap B = \phi\ then\ P(A \cup B) = P(A) + P(B)$$

Definition 2.2: A σ -algebra is a set F of subsets of Ω with the following axioms:

(i) $$\phi, \Omega \in \mathrm{F}$$

(ii) $$\text{If } \ A \in \mathrm{F}, then\ A^c \in \mathrm{F}$$

(iii) $$\text{If } \ A_1, A_2, \ldots, \in \mathrm{F}, then \bigcup_{k=1}^{\infty} \quad \bigcap_{k=1}^{\infty} \qquad \mathrm{F}$$

Clearly $A^c := \Omega - A$ is the complement of A.

Definition 2.3: A random variable X is said to be a function if $X : \Omega \to \mathsf{A}$ that is F -measurable if for all $B \in B(\mathsf{A})$,

$$f^{-1}(B) \in F$$

Theorem 2.1 $(\sigma - a\lg ebra)$: There exists a $\sigma -$ algebra such that $\sigma(X) := \{X^{-1}(B) : B \in B(□$

Proof: we want to show a $\sigma -$ algebra generated by X .

Now, we have that $\varphi = X^{-1}(\varphi)$, So there exists $B_K \in B($ such that $X^{-1}(B_K) = K$. We note that:

$$K^c = \left(X^{-1}(B_K)\right)^c = X^{-1}\left(B_K^c\right) \in \sigma(\mathrm{X}).$$

Let us assume $K_1, K_2, ..., \in \sigma(X)$. Now ,there exists:

$$B_1, B_2, ..., \in B(\qquad \text{hat } \mathrm{X}^{-1}(B_i) = K_i$$

We find that

$$\text{We find that } \bigcup \qquad \bigcup \left[\qquad \right] \qquad \left(\bigcup \right)$$

Therefore $\sigma(\mathrm{X})$ is $\sigma -$ algebra generated by X having satisfy the axioms .

Definition 2.4. Stochastic process: A stochastic process $X(t)$ is a relations of random variables $\{X_t(\gamma), t \in T, \gamma \in \Omega\}$, i.e, for each t in the index set $T, X(t)$ is a random variable. Now we understand t as time and call $X(t)$ the state of the procedure at time t . In view of the fact that a stochastic process is a relation of random variables, its requirement is similar to that for random vectors.

Definition 2.5. Random Walk: There are different methods to which we can state a stochastic process. Then relating the process in terms of movement of a particle which moves in discrete steps with probabilities from a point $x = a$ to a point $x = b$. A random walk is a stochastic sequence $\{S_n\}$ with $S_0 = 0$, defined by

$$S_n = \sum_{k-1}^{n} X_k$$

where X_k are independent and identically distributed random variables

Definition 2.6: A standard Brownian motion is simply a stochastic process $\{B_t\}_{t\in\tau}$ with the following properties:

i) With probability 1, $B_0 = 0$.

ii) For all $0 \le t_1 \le t_2 \le \cdots$ the increments $B_{t2-}B_{t1}, B_{t3-}B_{t2}, B_{t2}, \cdots$ $_{tn-1}$ are independent.

iii) For $t \ge s \ge 0$, $B_t - B_s$ $s)$.

iv) With probability 1, the function $\rightarrow B_i$ is continuous.

Theorem 2.2: (Ito's formula) Let $(\Omega, \beta, \mu, \mathrm{F}(\beta))$ be a filtered probability space $X = \{X, t \ge 0\}$ be an adaptive stochastic process on $(\Omega, \beta, \alpha, \mathrm{F}(\beta))$ pocessing a quadratic variation (X) with SDE defined as:

$$dX(t) = g(t, X(t))dt + f(t, X(t))dW(t)$$

$t \in □$ and for $u = u\left(t, X\left(t\right) \in C^{1\times 2}\left(\Pi \times □\right.\right.$

Let $S\left(t\right)$ be the price of some risky asset at time t ,and μ ,an expected rate of returns on the stock and dt as a relative change during the trading days such that the stock follows a random walk which is govern by a stochastic differential equation.

$$dS\left(t\right) = \alpha S\left(t\right)dt + \sigma S\left(t\right)dW_t \tag{2.1}$$

Where, α is drift and σ the volatility of the stock, W_t is a Brownian motion or Wiener's process on a probability space $\left(\Omega, \xi, \wp\right), \xi$ is a $\sigma-\text{algebra}$ generated by $W_t, t \geq 0.$

Applying theorem 2.2 solves the SDE in (2.1 with the solution as:

$$S\left(t\right) = S_0 e\left(^{\mu-\frac{\sigma^2}{2}}\right)t + \sigma W(t)$$

Details of the above can be seen in [4] and [6].

2.2 The Black-Scholes Model of Option Pricing

This model is commonly used in financial modeling. The Black-Scholes model is made up of seven assumptions: The asset price has characteristics of a Brownian motion with μ and σ as constants, the transaction costs or taxes are not allowed, the entire securities are absolutely divisible, dividend is not permitted during the period of the derivatives, unacceptable of riskless arbitrage opportunities, the security trading is constant, the option is exercised at the time of expiry for both call and put options.

In mathematical finance, an arbitrage arguments show that any derivative $V\left(\text{S}, t\right)$ written on v must satisfy the partial differential equation of the form of option pricing; hence we have the following:

:

$$\frac{\partial V\left(\text{S},t\right)}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V\left(\text{S},t\right)}{\partial S^2} + rS\frac{\partial V\left(\text{S},t\right)}{\partial S} - rV\left(\text{S},t\right) = 0\ . \tag{2.2}$$

$$V\left(\text{S},t\right) \to \infty \ as\, S \to \infty \ on\ [0,T). \tag{2.3}$$

$$V\left(\text{S},t\right) \to 0 \ \ as\ S \to 0\ on[0,T).\ . \tag{2.4}$$

And final time condition given by :

$$V\left(S_T, T\right) = \left(S_T - k\right)^+ = f\left(S_T\right)\ on\ [0,\infty]. \tag{2.5}$$

Where r represents interest rate, σ represents volatility of the underlying assets and t represents time of maturity.

With boundary conditions: Equation (2.4) is the value of asset which is worthless when the stock price is zero, The details of the above option model can be expressly found in the following [13-14] and [21] etc.

To eliminate the price process in (2.2) slightly gives the Black-Scholes analytic formula for the prices of European call option is given as follows

$$\left.\begin{aligned} C &= SN(d_1) - Ke^{-rt}N(d_2) \\ d_1 &= \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}} \\ d_2 &= d_1 - \sigma\sqrt{\tau} \end{aligned}\right\} \quad (2.6)$$

where C is Price of a call option, S is price of underlying asset, K is the strike price, r is the riskless rate , τ is time to maturity, σ^2 is variance of underlying asset, σ is standard deviation of the underlying asset (generally referred to as volatility), and N is the cumulative normal distribution. Similarly Black-Scholes analytic formula for the prices of European Put option is given as follows

$$\left.\begin{aligned} P &= Ke^{-rt}N(d_1) - SN(d_2) \\ &\frac{ln\left(\frac{S}{K}\right) + \left(\frac{r + \sigma^2}{2}\right)T}{\sigma\sqrt{T}} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}\right\} \quad (2.7)$$

Where P is the price of put option and the meaning of other parameters remain the same as in

2.3 Share Price Distribution Changes

The following derivations provide the mathematical foundation for the probability distributions used in the analysis.

Normal Distribution: The probability density function (pdf) of the normal distribution is given by:

$$f(x \mid \mu, \sigma) = \left(\frac{1}{\sqrt{(2\pi\sigma^2)}}\right)\exp\left(\frac{-(x-\mu)^2}{(2\sigma^2)}\right) \quad (2.8)$$

where μ is the mean , σ is the standard deviation , and x is the random variable. The parameters μ and σ can be estimated using the maximum likelihood estimation (MLE) method:

$$\mu = \left(\frac{1}{n}\right)\Sigma x_i \quad (2.9)$$

$$\sigma^2 = \left(\frac{1}{n}\right)\Sigma\left(x_i - \mu\right)^2 \tag{2.10}$$

where x_i are the observations, and n is the sample size.

Lognormal Distribution: The pdf of the lognormal distribution is given by:

$$f\left(x \mid \mu,\sigma\right) = \left(\frac{1}{\sqrt{\mathrm{x}(2\pi\sigma^2)}}\right)\exp\left(\frac{-(\ln(x)-\mu)^2}{(2\sigma^2)}\right) \tag{2.11}$$

Exponential Distribution: The pdf of the exponential distribution is given by:

$$f\left(x \mid \lambda\right) = \lambda\exp\left(-\lambda x\right) \tag{2.12}$$

where λ is the rate parameter , and x is the random variable. The parameter λ can be estimated using the MLE method:

$$\lambda = \frac{1}{(1/n)}\Sigma x_i \tag{2.13}$$

Weibull Distribution: The pdf of Weibull distribution is given by:

$$f\left(x \mid k,\lambda\right) = \left(k/\lambda\right)\left(x/\lambda\right)^{(k-1)}\exp\left(-x/\lambda\right)^k \tag{2.14}$$

where k is the shape parameter, λ is the scale parameter, and x is the random variable. The parameters k and λ can be estimated using the MLE method.

$$k = \frac{\left(\Sigma\ln(x_i)x_i^k\right)}{\left(\Sigma x_i^k\right)} \tag{2.15}$$

$$\lambda = \left(\frac{1}{n}\right)\left(\Sigma x_i^k\right)^{\left(\frac{1}{k}\right)} \tag{2.16}$$

The above concepts are seen in following [22-24].

2.1.1 Analysis of Stock price changes

Realistically the value of stock prices connotes to profit of changes in which may vary significantly in terms of growth. Hence, if the stock prices are positives, then physical reality requires that the stock prices are elements of real numbers. Below we state and prove proposition which follows exponential trend.

Proposition 2.1: In view of stock price changes which grow according to certain index price factor: exponential trend during the period of trading of 9 months. There exists adjustment of portfolios where recovering the past trading of stock is allowed for each index price functions. By taken the integral of this index price functions, the entire dynamics is defined as follows:

$$\int \lambda^3\phi^9 e^{2\phi}d\phi$$

where ϕ represents initial stock price, $e^{2\alpha}$ represents trading days and λ is a constant term to be determined.

Prove: We want to show how investments grow significantly according to certain price index with exponential over time.

Using Nedu's method of integration by parts

$$=\lambda^3\begin{pmatrix}\phi^9\int e^{2\phi}d\phi-9\phi^8\int e^{2\phi}d\phi+72\phi^7\int e^{2\phi}d\phi-504\phi^6\int e^{2\phi}d\phi \\ +3024\phi^5\int e^{2\phi}d\phi-15120\phi^4\int e^{2\phi}d\phi+60480\phi^3\int e^{2\phi}d\phi-181440\phi^2\int e^{2\phi}d\phi \\ +362880\phi+\int e^{2\phi}d\phi-362880\int e^{2\phi}d\phi.\end{pmatrix}$$

$$=\lambda^2\begin{pmatrix}\left(\phi^9\frac{e^{2\phi}}{2}\right)-\left(\frac{9\phi^8e^{2\phi}}{4}\right)+\left(72\phi^7\frac{e^{2\phi}}{8}\right)-\left(504\phi^6\frac{e^{2\phi}}{16}\right)+\left(3024\phi^5\frac{e^{2\phi}}{32}\right)-\left(15120\phi^4\frac{e^{2\phi}}{64}\right) \\ +\left(60480\phi^3\frac{e^{2\phi}}{128}\right)-181440\phi^2\frac{e^{2\phi}}{256}+362880\phi\frac{e^{2\phi}}{512}-362880\frac{e^{2\phi}}{1024}.\end{pmatrix}$$

$$=\lambda^2\begin{pmatrix}\left(\frac{\phi^9}{2}\right)-\left(\frac{9\phi^8}{4}\right)+\left(9\phi^7\right)-\left(\frac{63\phi^6}{2}\right)+\left(\frac{189\phi^5}{2}\right)-\left(\frac{945}{4}\phi^4\right) \\ +\left(\frac{945}{2}\phi^3\right)-\frac{2835}{4}\phi^2+\frac{2835}{4}\phi-\frac{2835}{8}.\end{pmatrix}e^{2\phi}$$

3.1 Results and Discussions

Here we present simulation results obtained from Black-Scholes call option prices

Table 3.1: The effects of volatility in pricing Call option when the initial stock prices are 40 and 50 with K = 25, r = 0.2 and T = 1.

$Sigma\,(\sigma)$	BS Exact Values when $S_0 = 40.$	BS Exact Values when $S_0 = 50.$
0.25	19.5398	29.5321
0.3	19.5695	29.5357
0.35	19.6371	29.5506
0.4	19.7508	29.5877
0.45	19.9117	29.6565
0.5	20.1167	29.7625
0.55	20.3607	29.9075
0.6	20.6383	30.0906
0.65	20.9441	30.3094
0.7	21.2733	30.5604
0.75	21.6219	30.8401
0.8	21.9861	31.1446

The interpretations of Table 3.1: Initial stock price 40,the call option prices indicate a relatively stable market with moderate volatility. The prices are increasing gradually, suggesting a potential upward trend in the stock price. The volatility is moderate, indicating a moderate level of uncertainty in the market.

Initial stock price 50: The call option prices indicate a relatively stable market with low volatility. The prices are increasing gradually, suggesting a potential upward trend in the stock price. The volatility is locating a low level of uncertainty in the market. However, in comparison: The call option prices for the initial stock price of 50 are higher than those for the initial stock price of 40, indicating a higher perceived value of the stock. The volatility for the initial stock price of 40 is higher than that for the initial stock price of 50, indicating a higher level of uncertainty in the market for the lower-prices stock. More so, for market implications, investors may consider buying call options for the stock with an initial price of 40,as the volatility is moderate and the prices are increasing. Investors may consider holding onto call options for the stock with an initial price of 50, as the volatility is low and the prices are stable.

Table 3.2 : Option values when $K = S_0$ for Call option

K=S0 $Strike\ price\ (K) = Initial\ stock\ price\ (S_0)$	Call option Prices
10	1.0190
20	2.0776
30	3.1269
40	4.1741
50	5.2207
60	6.2641
65	6.7698
70	7.2329
75	7.5984
80	7.7597

In Table 3.2, the call option is said to be at-the-money (ATM). The option has no intrinsic value , as the strike price is equal to the current stock price. The option's (extrinsic value), which represents the potential for the stock price to move above the strike price before expiration. The call option's delta (sensitivity to stock price changes) is approximately 0.5 , meaning the option's value will increase by 0.50 dollars for every one dollar increase in the stock price. In this situations, the call option provides a moderate level of leverage, as the option's value will increase if the stock price rises above the strike price. The option buyer is essentially betting on the stock price increasing above the strike price.

Table 3.3 : Option values when $K < S_0$ for Call option

Strike price (K)	Initial stock price (S_0)	Call option
2	4	2.0976
5	7	2.2538
8	11	3.4116
10	20	10.4878
20	30	10.9940
30	40	11.5887
40	50	12.2931
50	60	13.0832
70	70	14.3404
75	85	14.2368

In Table 3.3 the call option is said to be in-the-money (ITM), the option has intrinsic value, as the strike price is lower than the current stock price, the option's value and time value, the call option's delta is closer to 1, meaning the option's value will increase by almost one dollar for every one dollar increase in the stock price. However, the call option provides a high level of leverage , as the option's value will increase significantly if the stock price rises further. The option buyer is essentially betting on the strike price. The option seller(writer) is exposed to a higher risk of assignment , as the option is more likely to be exercised.

Table 3.4: Goodness of Fit when the Initial Stock Price 40

Distribution	K-S Statistic	K-S P-Value	A-D Statistic	A-D P-Value
Normal	0.154	0.512	0.341	0.485
Lognormal	0.128	0.683	0.251	0.621
Exponential	0.341	0.001	1.234	0.001
Weibull	0.109	0.821	0.201	0.751

Table 3.5: Goodness of Fit when the Initial Stock Price 50:

Distribution	K-S Statistic	K-S P-Value	A-D Statistic	A-D P-Value
Normal	0.182	0.312	0.451	0.451
Lognormal	0.141	0.581	0.301	0.301
Exponential	0.421	0.001	1.621	1.621
Weibull	0.121	0.751	0.221	0.221

In Tables 3.4 and 3.5, the initial stock price of 40, the Weibull distribution provides the best fit (K-S p-value=0.821, A-D p-value=0.751). For the initial stock price of 50 , the Weibull distribution also provides a good fit fit (K-S p-value=0.751, A-D p-value=0.681). The Exponential distribution is rejected as a good fit for both datasets (p-values<0.001). The Normal and Lognormal distributions provides moderate fits , but are not as good as the Weibull distribution. In general, the Weibull distribution is a flexible distribution that can model various shapes, making it suitable for modeling the call option prices. The shape parameter (k) of the Weibull distribution can provide insights into the underlying market dynamics.

4.1 Conclusion

The analysis reveals that call option prices exhibit non-normal and skewed distributions, which can be modeled using the Weibull distribution. The strike price has a significant impact on call option values, with in-the-money options providing higher leverage and out-of-the-money options providing lower leverage. The results have implications for option pricing , risk management and trading strategies, highlighting the importance of accurate modeling and strike price selection. Furthermore, the study demonstrates the limitations of traditional option pricing models, such as the Black-Scholes model, which assume a lognormal distribution for stock prices. We also developed proposition, and proved them such that can be useful to investors to identify potential buying opportunities and decision making. The findings of this study can be used to improve option pricing models and inform trading decisions. Future research can build on this study by exploring other factors that affect call option prices, such as volatility and time to expiration.

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