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Thermodynamic Properties Of Schwarzschild–De Sitter And Kerr–De Sitter Black Holes In Extended Phase Space

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ABSTRACT

The thermodynamic behavior of black holes in spacetimes with a positive cosmological constant remains an important subject in gravitational physics due to the presence of multiple horizons and the absence of global thermal equilibrium. In this work, we investigate the thermodynamic properties of Schwarzschild–de Sitter and Kerr–de Sitter black holes within the framework of extended black hole thermodynamics, where the cosmological constant is interpreted as a thermodynamic pressure. The study analyzes the geometric origin of horizon temperature, entropy, and surface gravity in de Sitter black hole spacetimes and examines the effects of rotation on thermodynamic behavior. For Schwarzschild–de Sitter spacetime, the coexistence of black hole and cosmological horizons leads to distinct temperatures and entropies, while Kerr–de Sitter spacetime introduces additional structure through angular momentum and frame dragging. The extended phase space formalism is used to reinterpret the first law of black hole thermodynamics in the presence of a cosmological constant. The results demonstrate that de Sitter black holes differ fundamentally from asymptotically flat and anti–de Sitter systems due to their inherently non-equilibrium thermodynamic nature.

Keywords: black holes, thermodynamic behavior, spacetimes

1. INTRODUCTION

Black hole thermodynamics represents one of the most significant developments in modern theoretical physics, providing deep connections between gravitation, quantum mechanics, and statistical physics (Hawking & Ellis, 1973; Gibbons & Hawking, 1977). The discovery that black holes possess entropy and emit thermal radiation established the idea that spacetime horizons obey thermodynamic laws analogous to those of ordinary physical systems (Gibbons & Hawking, 1977). These developments have motivated extensive investigations into the thermodynamic interpretation of gravitational systems in a wide variety of spacetime geometries (Carroll, 2001; Frassino et al., 2023).

The cosmological constant Λ has re-emerged as a central component of gravitational physics following observational evidence for the accelerated expansion of the universe (Bull et al., 2016; Planck Collaboration, 2020). In Einstein's theory of general relativity, a non-zero cosmological constant modifies the large-scale curvature of spacetime and gives rise to maximally symmetric geometries such as de Sitter and anti–de Sitter spacetimes (Einstein, 1917; Weinberg, 1972). In the case of a positive cosmological constant, the resulting de Sitter geometry possesses a cosmological horizon associated with temperature and entropy, thereby introducing additional thermodynamic structure (Anninos, 2012; Gibbons & Hawking, 1977).

Black holes embedded in de Sitter backgrounds exhibit thermodynamic properties that differ substantially from those of asymptotically flat black holes (Bousso, 2002; Stuchlík & Hledík, 1999). Schwarzschild–de Sitter spacetime contains both a black hole event horizon and a cosmological horizon, each characterized by distinct temperatures and entropies (Gibbons & Hawking, 1977; Stuchlík & Hledík, 1999). Since these temperatures are generally unequal, the spacetime does not admit a state of global thermal equilibrium (Gomberoff & Teitelboim, 2003; Urano et al., 2009). Kerr–de Sitter spacetime introduces additional complexity through rotation, which modifies the horizon structure and generates ergoregions associated with frame dragging (Akcay & Matzner, 2011; Carter, 1968). Recent developments in extended black hole thermodynamics reinterpret the cosmological constant as a thermodynamic pressure (Dolan, 2014; Kubizňák & Mann, 2012). This approach enlarges the thermodynamic phase space and allows gravitational systems to be analyzed using methods analogous to ordinary thermodynamics. Within this framework, black hole mass is interpreted as thermodynamic enthalpy, while the conjugate thermodynamic volume emerges naturally from the first law.

This work investigates the thermodynamic properties of Schwarzschild–de Sitter and Kerr–de Sitter black holes within the extended phase space formalism. Particular attention is given to horizon temperature, entropy, surface gravity, and the role of the cosmological constant in modifying thermodynamic behavior.

2. Einstein Field Equations and de Sitter Geometry

Einstein's field equations with a cosmological constant are given by:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}. \qquad 1$$

Einstein's cosmological modification of the field equations introduced the cosmological constant as a geometric contribution to spacetime curvature (Einstein, 1917; Weinberg, 1972).

Where $G_{\mu\nu}$ is the Einstein tensor describing spacetime curvature, Λ is the Cosmological constant, $g_{\mu\nu}$ is the Metric tensor of spacetime, $T_{\mu\nu}$ is the stress–energy tensor, μ,ν are the Spacetime coordinate indices, 8π is the Geometrized gravitational coupling constant ($G = c = 1$)

In a vacuum,

$$T_{\mu\nu} = 0,$$

so that:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \qquad 2$$

Equivalently,

$$R_{\mu\nu} = \Lambda g_{\mu\nu}. \qquad 3$$

Maximally symmetric vacuum solutions of Einstein's equations with positive cosmological constant correspond to de Sitter spacetime (Hawking & Ellis, 1973; Anninos, 2012). Here $R_{\mu\nu}$ is the Ricci curvature tensor, Λ is cosmological constant, $g_{\mu\nu}$ is the Metric tensor.

These equations admit that the de Sitter spacetime as a maximally symmetric vacuum solution for $\Lambda > 0$.

The static form of the de Sitter metric is:

$$ds^2 = -\left(1-\frac{\Lambda r^2}{3}\right) dt^2 + \left(1-\frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2 d\Omega^2. \qquad 4$$

Where ds^2 is a spacetime interval, t is the time coordinate, r is the radial coordinate, Λ is cosmological constant, $d\Omega^2$ is the angular line element on the unit 2-sphere, $d\theta^2 + sin^2\theta d\phi^2$ is the explicit form of $d\Omega^2$. The de Sitter metric describes a spacetime of constant positive curvature and possesses a cosmological horizon associated with thermal properties (Gibbons & Hawking, 1977).

The corresponding cosmological horizon occurs at:

$$r_c = \sqrt{\frac{3}{\Lambda}}. \qquad 5$$

Where r_c is the Cosmological horizon radius, Λ is a positive cosmological constant.

The Ricci scalar for de Sitter spacetime is:

$$R = 4\Lambda. \qquad 6$$

Where R is the Ricci scalar curvature and Λ the cosmological constant.

These results show that the cosmological constant acts as a direct measure of global spacetime curvature.

3. Thermodynamics of Schwarzschild–de Sitter Black Holes

The Schwarzschild–de Sitter metric is:

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2, \qquad 7$$

Where,

$$f(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}. \qquad 8$$

Where $f(r)$ is the Metric function, M is the black hole mass, r is the radial coordinate, Λ is the Cosmological constant. The Schwarzschild–de Sitter solution represents a static black hole embedded within an expanding de Sitter universe (Kottler, 1918; Stuchlík & Hledík, 1999).

The horizons are obtained from:

$$f(r) = 0. \qquad 9$$

Here r is the horizon radius, depending on the values of $M$and Λ, the spacetime admits both black hole and cosmological horizons (Bousso, 2002).

For a positive Λ, the spacetime generally contains a black hole horizon r_b, and a cosmological horizon r_c.

3.1 Surface Gravity

The surface gravity associated with a horizon r_h is:

$$\kappa = \frac{1}{2}f'(r_h). \tag{10}$$

Where κ is Surface gravity, $f'(r_h)$ is the Derivative of metric function evaluated at the horizon, r_h is the Generic horizon radius

Evaluating the derivative k becomes:

$$\kappa = \frac{1}{2}\left(\frac{2M}{r_h^2} - \frac{2\Lambda r_h}{3}\right).$$

3.2 Hawking Temperature

The Hawking temperature is:

$$T = \frac{\kappa}{2\pi}. \tag{11}$$

T is the Hawking temperature, k is the surface gravity and π is a mathematical constant, the temperature associated with black hole horizons arises from quantum field theoretic effects in curved spacetime (Hawking & Ellis, 1973).

Thus,

$$T = \frac{1}{4\pi} \left| \frac{1}{r_h} - \Lambda r_h \right|.$$

Since $r_b \neq r_c$,

$$T_b \neq T_c.$$

This implies that Schwarzschild–de Sitter spacetime does not possess global thermodynamic equilibrium.

3.3 Entropy

Black hole entropy satisfies the Bekenstein–Hawking relation:

$$S = \frac{A}{4}, \tag{12}$$

The Bekenstein–Hawking entropy relation establishes the proportionality between entropy and horizon area (Gibbons & Hawking, 1977). Here, S is the Blackhole entropy and A the Horizon area.

Where

$$A = 4\pi r_h^2. \tag{13}$$

Hence,

$$S = \pi r_h^2. \tag{14}$$

Both the black hole and cosmological horizons contribute independent entropy terms.

4. Thermodynamics of Kerr–de Sitter Black Holes

Kerr–de Sitter spacetime generalizes the Schwarzschild–de Sitter solution by including angular momentum.

The radial metric function is:

$$\Delta_r = (r^2 + a^2)\left(1 - \frac{\Lambda r^2}{3}\right) - 2Mr. \tag{15}$$

Where Δ_r is the radial horizon function, r is the Radial coordinate, a is the rotation parameter (specific angular momentum), M is the black hole mass and Λ is the cosmological constant. Kerr–de Sitter spacetime extends the Schwarzschild–de Sitter geometry by incorporating rotation and frame-dragging effects (Carter, 1968; Akcay & Matzner, 2011).

Horizons are determined from:

$$\Delta_r = 0. \qquad 16$$

These roots determine the Cauchy horizon, event horizon and cosmological horizon (Grenzebach et al., 2014).

4.1 Horizon Structure

Depending on the parameters (M, a, Λ), the spacetime may contain; an inner Cauchy horizon, an outer event horizon, and a cosmological horizon.

4.2 Angular Velocity

The angular velocity at the horizon is:

$$\Omega = \frac{a}{r_h^2 + a^2}. \qquad 17$$

Where Ω is the angular velocity of the horizon, a the rotation parameter, r_h is the horizon radius.

This rotation modifies the thermodynamic behavior by coupling angular momentum to horizon geometry.

4.3 Temperature

The Hawking temperature for Kerr–de Sitter spacetime is:

$$T = \frac{\Delta_r'}{4\pi(r_h^2 + a^2)}. \qquad 18$$

Where T is the Hawking temperature, Δ_r' is the derivative of Δ_r, r_h is the horizon radius and a is the rotation parameter. Rotation modifies the surface gravity and consequently alters the thermodynamic properties of the spacetime (Akcay & Matzner, 2011).

The presence of angular momentum reduces the temperature relative to the non-rotating case and introduces additional parameter dependence, as shown in Figure 1 below.

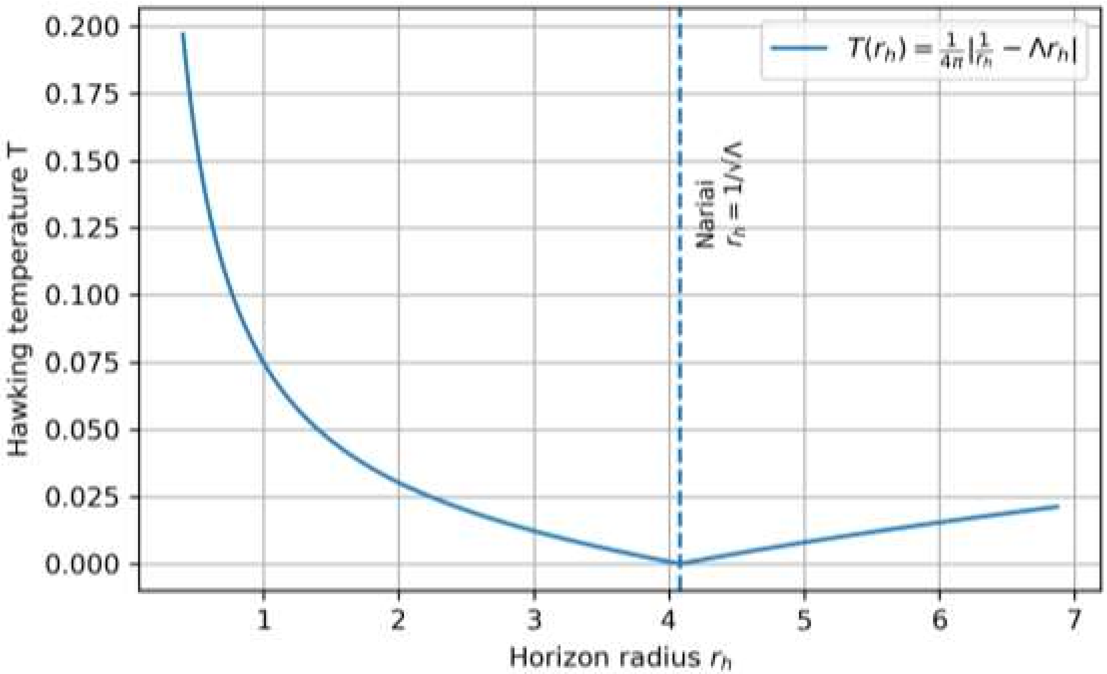

Figure 1: Hawking Temperature Tas a Function of Horizon Radius r_h for Schwarzschild-de Sitter Black Holes.

4.4 Entropy

In SdS and KdS spacetimes, the existence of multiple horizons complicates the phase structure as shown in Figure 2, which is a plot of KdS entropy against horizontal radius), but the local thermodynamic description of each horizon remains valid.

The entropy is:

$$S = \frac{A}{4},$$

where the horizon area becomes:

$$A = \frac{4\pi(r_h^2 + a^2)}{\Xi},$$

with

$$\Xi = 1 + \frac{\Lambda a^2}{3}. \qquad 19$$

Hence,

$$S = \frac{\pi(r_h^2 + a^2)}{\Xi}. \qquad 20$$

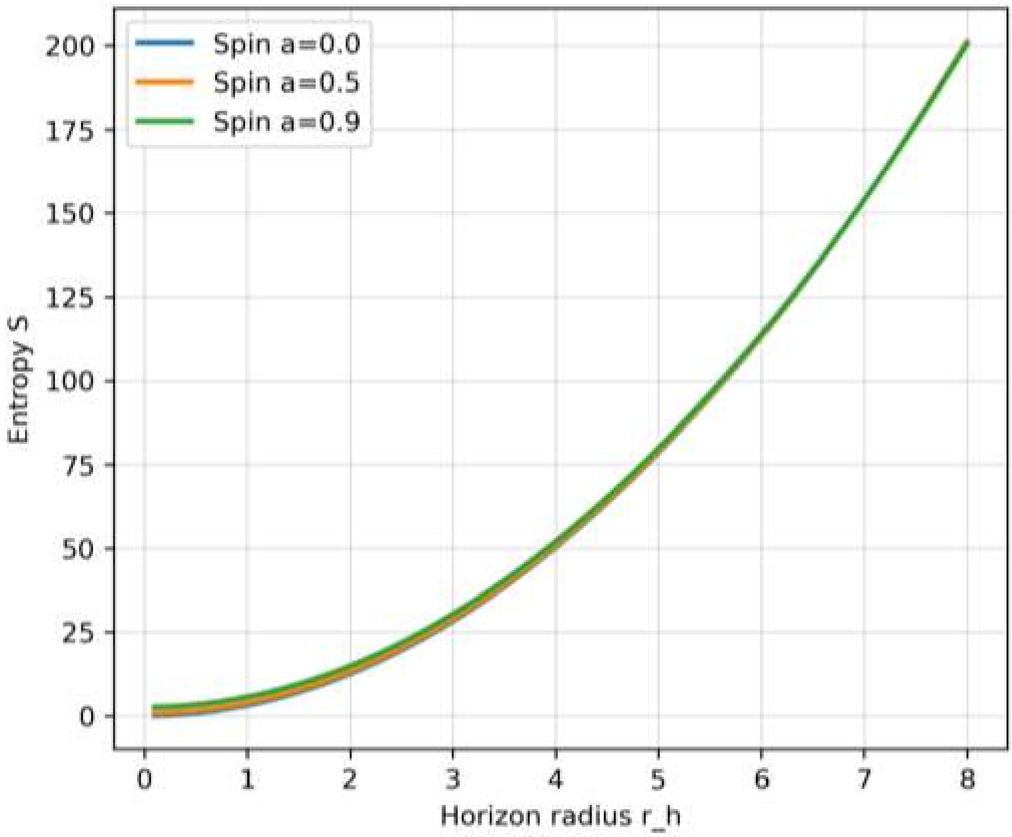

Figure 2: Entropy S versus Horizon Radius r_h for Kerr–de Sitter Black Holes (Different spins).

5. Extended Phase Space Thermodynamics

In extended black hole thermodynamics, the cosmological constant is interpreted as thermodynamic pressure:

$$P = \frac{\Lambda}{8\pi}. \qquad 27$$

The conjugate thermodynamic volume is:

$$V = \left(\frac{\partial M}{\partial P}\right)_S. \qquad 28$$

The generalized first law becomes:

$$dM = TdS + \Omega dJ + VdP. \qquad 29$$

Within this framework, black hole mass is interpreted as enthalpy rather than internal energy.

The extended phase space approach provides a natural thermodynamic interpretation for the cosmological constant and enables the study of pressure–volume effects in gravitational systems.

6. DISCUSSION

The thermodynamic properties of de Sitter black holes differ fundamentally from those of asymptotically flat and anti–de Sitter black holes. The presence of multiple horizons with distinct temperatures prevents the existence of global thermal equilibrium and introduces inherently non-equilibrium thermodynamic behavior.

In Schwarzschild–de Sitter spacetime, the black hole and cosmological horizons contribute independent thermodynamic quantities. The resulting system cannot be described by a single equilibrium temperature. Kerr–de Sitter spacetime further enriches this structure through rotational effects, ergoregions, and modified horizon geometry.

The extended phase space formalism demonstrates that the cosmological constant functions not only as a geometric parameter but also as a thermodynamic variable analogous to pressure. This interpretation provides deeper insight into the thermodynamic role of spacetime curvature.

7. CONCLUSION

This work has investigated the thermodynamic properties of Schwarzschild–de Sitter and Kerr–de Sitter black holes within the framework of extended black hole thermodynamics. The analysis demonstrates that: de Sitter black holes possess multiple horizons with distinct temperatures, global thermal equilibrium is generally absent, rotation significantly modifies thermodynamic behavior, and the cosmological constant admits a consistent interpretation as thermodynamic pressure. These findings highlight the rich thermodynamic structure of Λ-dominated spacetimes and reinforce the importance of non-equilibrium approaches in de Sitter black hole thermodynamics.

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