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Machine-Learning-Enhanced Multivariate GARCH Models for modeling Volatility Spillover in Nigeria Equity Markets

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ABSTRACT

This research examines the changing volatility spillovers, non-linear relationships, and long-term equilibrium between the Nigerian All Share Index (ASI) and Market Capitalization (MC) utilizing the empirical structure of Machine-Learning-Enhanced Multivariate GARCH Models for analyzing volatility spillover in the Nigerian equity markets. The analysis is based on 1,799 monthly secondary data points collected from January 1996 to June 2024 sourced from the Central Bank of Nigeria, with the data being processed through Python libraries (arch, statsmodels, tensorflow, pytorch). Preliminary analyses reveal non-normal distribution through Jarque-Bera tests, stationarity in returns confirmed by the Augmented Dickey-Fuller and Phillips-Perron tests, and the presence of conditional heteroskedasticity indicated by ARCH-LM tests. While the Vector Error Correction (VEC) methodology shows a long-term equilibrium, reflected by an ASI adjustment speed coefficient of 66.3% for each period, the eigenvalues of the companion matrix (with moduli of 18.16 and 14.63) highlight dynamic instability in linear models. To address this constraint, three hybrid models—VEC–LSTM–DCC, VEC–ANN–DCC, and VEC–SVR–DCC—were evaluated. An examination of the structural parameter weights indicates a significant one-way volatility spillover from Market Capitalization to the All Share Index. Within the VEC–LSTM–DCC framework, the initial input weights (W_0) demonstrate that Market Capitalization plays a more substantial role (showing positive weights as high as 0.2084) on memory cell activations compared to ASI, which has negative weights close to -0.0373. The assessment of predictive capabilities shows consistent reliability throughout all hybrid models, where the VEC–ANN–DCC framework stands out with its superior effectiveness marked by the lowest Mean Squared Error (2.199989×10^7), Root Mean Squared Error (4.690404×10^3) and Mean Absolute Error (3.870982×10^2). On the other hand, the VEC–SVR–DCC framework experienced somewhat greater error estimates ($MSE = 2.202199 \times 10^7$). The findings suggest that the combination of artificial neural networks with Dynamic Conditional Correlation models yields the best accuracy in effectively capturing asymmetric, non-linear volatility transmission among indicators in the Nigerian equity market.

Keywords: Machine, Learning, -Enhanced, Multivariate, GARCH, Models, Volatility & Spillover

INTRODUCTION

The modelling of volatility spillovers in developing financial markets continues to be a significant focus in empirical finance, especially in countries like Nigeria where stock markets react strongly to macroeconomic disturbances, variations in oil prices, fluctuations in exchange rates, and speculative capital movements. Initial empirical methodologies heavily depended on linear models such as Vector Autoregression (VAR) and Vector Error Correction Models (VECM) to reflect dynamic associations among financial factors. However, these models have a notable shortcoming in that they fail to effectively depict nonlinear relationships, regime-specific volatility, and time-dependent conditional heteroskedasticity that is typical of financial returns.

Specifically, while the VECM structure is successful in representing long-term equilibrium connections among cointegrated elements, it presumes linear adjustment trajectories toward equilibrium and balanced error-correction mechanisms. This presumption can become restrictive in financial markets where adjustment rates differ among regimes and where shocks influence outcomes in an asymmetric manner (Deebom & Tuaneh,2019).

Research evidence indicates that financial time series in Nigeria display features like volatility clustering, leptokurtosis, and structural breaks, which contravene the principles underlying linear cointegration models (Atoi, 2014; Salisu, Isah & Assandri, 2019). As a result, approaches based on VECM frequently fail to adequately encompass dynamic fluctuations and nonlinear interactions across markets (Deebom, Bharat & Inamete, 2020, and Deebom,2025).

To overcome these challenges, multivariate GARCH (MGARCH) models—particularly the BEKK-GARCH and DCC-GARCH types—have been extensively utilized to analyze volatility transmission and shock spillovers across various financial markets. Investigations by Emenike (2014) and Ali et al. (2021) reveal significant two-way volatility spillovers between Nigerian stock and foreign exchange markets when employing BEKK-type models, underscoring the interconnected nature of financial sectors. Likewise, research by Salisu, Isah, and Assandri (2019) presents compelling evidence of robust inter-market shock transmission between stock and money markets in Nigeria, reinforcing the necessity for multivariate volatility analysis.

Notwithstanding these advancements, conventional MGARCH frameworks remain substantially parametric and impose inflexible functional relationships on volatility mechanisms. Consequently, they can struggle to adequately capture intricate nonlinear interactions, complex relationships, and shifting market dynamics. Empirical research has shown that financial markets—particularly in emerging economies—display regime-switching characteristics, asymmetric reactions to shocks, and nonlinear spillover effects that standard BEKK or CCC specifications cannot fully elucidate. Moreover, even hybrid econometric extensions that incorporate structural breaks or asymmetric error distributions have demonstrated limited effectiveness in consistently enhancing forecasting accuracy during volatile market environments.

Recent progress in machine learning has provided more adaptable models capable of addressing these limitations. Frameworks like Artificial Neural Networks (ANN), Support Vector Regression (SVR), and Long Short-Term Memory (LSTM) networks have been combined with MGARCH structures to improve the representation of nonlinear relationships and dynamic volatility clustering. Research into hybrid ARIMA–GARCH and machine learning–GARCH models within Nigerian equity markets indicates improved prediction accuracy and a better capacity to model volatility persistence, as compared to traditional econometric methods (Ekpete et al., 2026). Nonetheless, despite increasing interest, the use of machine-learning-enhanced multivariate GARCH models for analyzing volatility spillovers in Nigerian equity markets is still inadequate.

Considering this situation, this research is driven by the necessity to address the shortcomings of conventional linear and solely econometric volatility models through the creation of a Machine-Learning-Enhanced Multivariate GARCH framework designed for analyzing volatility spillovers in the equity markets of Nigeria. More specifically, this study aims to combine the benefits of MGARCH models in recognizing conditional heteroskedasticity and transmission of volatility with the nonlinear learning abilities of machine learning techniques. The distinct aims of this study include investigating the presence

and direction of volatility spillovers among various indicators of the Nigerian equity market; assessing how traditional VECM and standard MGARCH models fall short in capturing nonlinear dependencies across markets; and creating and evaluating the predictive capabilities of hybrid Machine-Learning-Enhanced MGARCH models to manage dynamic volatility transmission more effectively compared to standard methods. Through the pursuit of these aims, the research enhances the evolving methodologies in financial econometrics, especially in emerging markets where the dynamics of volatility are intricate, nonlinear, and subject to structural instability

METHODOLOGY

The research approach begins with the collection of 1,799 monthly secondary data points from January 1996 to June 2024 sourced from the Central Bank of Nigeria's website, emphasizing monthly returns from the Nigerian Capital Markets, volatility, and various macroeconomic indicators. The processing of data and statistical modeling is carried out using Python, employing numerical and econometric libraries like `arch`, `statsmodels`, `numpy`, `scipy`, `scikit-learn`, `tensorflow`, and `pytorch`. The analytical process initiates with an initial data assessment, converting raw price sequences into logarithmic returns and volatility for both the All Share Index and Market Capitalization to remove outliers and achieve stationarity. Time series visualizations are generated to examine trends, seasonal effects, and cyclical behaviors throughout the sampled period.

Subsequently, descriptive statistics and the Jarque-Bera test are utilized to examine normality, where any deviations from normal distribution validate the use of Multivariate GARCH models that incorporate flexible error distributions. Correlation analysis is conducted to explore pairwise linear connections. To assess stationarity and establish integration orders, the Augmented Dickey-Fuller and Phillips-Perron unit root tests are performed under random walk conditions, with and without drift and trend, and non-stationary series are differenced if necessary. Residuals from the mean equations undergo testing for conditional heteroskedasticity using the ARCH Lagrange Multiplier test, relying on the nR^2 test statistics compared to the chi-square distribution.

Following this, an unrestricted Vector Autoregressive (VAR) model is estimated to capture the dynamic feedback interactions between the All-Share Index and Market Capitalization, determining the optimal lag length by minimizing the Akaike Information Criterion. Finally, Johansen cointegration tests—utilizing Trace and Maximum Eigenvalue likelihood ratio statistics—are performed to ascertain the presence of a long-run equilibrium relationship, with the identification of cointegration leading to the application of Vector Error Correction modeling to assess short-run adjustment rates toward long-run equilibrium

Similarly, the models used for this studied include VEC–LSTM–DCC, VEC–ANN–DCC, and VEC–SVR–DCC. These models preserve the cointegration and non-linear mean estimation phases (VECM + ML residuals) but substitute the covariance specification with a two-stage time-varying correlation process. For the Dynamic Conditional Correlation (DCC-GARCH) Structure, Let the residual error vector $\boldsymbol{\eta}_t$ derived from the respective hybrid mean model (VEC-LSTM, VEC-ANN, or VEC-SVR) be expressed as:

$$\boldsymbol{\eta}_t = \mathbf{H}_t^{1/2}\mathbf{z}_t, \quad \mathbf{z}_t \sim \text{i.i.d.}\ (\mathbf{0}, \mathbf{I}_2)$$

Under the DCC framework, the (2×2) conditional covariance matrix $\mathbf{H}_t$ is decomposed into

conditional standard deviations and a time-varying correlation matrix:

$$\mathbf{H}_t = \mathbf{D}_t\mathbf{R}_t\mathbf{D}_t$$

where:$\mathbf{D}_t = \text{diag}\left(h_{ASI,t}^{1/2}, h_{MCAP,t}^{1/2}\right)$ is a diagonal matrix containing the time-varying standard deviations derived from univariate GARCH models applied to each series. $\mathbf{R}_t$ is the time-varying conditional correlation matrix of dimension (2×2):

$$\mathbf{R}_t = \begin{bmatrix} 1 & \rho_{ASI,MCAP,t} \\ \rho_{MCAP,ASI,t} & 1 \end{bmatrix}$$

Similarly, Univariate GARCH Volatility Specifications (D_t) start Each individual asset variance ($i \in \{ASI, MCAP\}$) is modeled using a standard GARCH (p, q) process on the hybrid residuals $\eta_{i,t}$:

$$h_{i,t} = \omega_i + \sum_{j=1}^{q} \alpha_{i,j}\, \eta_{i,t-j}^2 + \sum_{k=1}^{p} \beta_{i,k}\, h_{i,t-k}$$

The standardized residuals are then computed as:

$$\nu_{i,t} = \frac{\eta_{i,t}}{\sqrt{h_{i,t}}}, \quad \text{such that} \quad \boldsymbol{\nu}_t = \left[\nu_{ASI,t}, \nu_{MCAP,t}\right]^T$$

For Dynamic Correlation Process (R_t), The standardized residuals $\boldsymbol{\nu}_t$ are used to drive the proxy process $\mathrm{Q}_t = \left(q_{ij,t}\right)$, which dictates the evolution of dynamic correlations:

$$\mathbf{Q}_t = \left(1 - \sum_{i=1}^{M} \alpha_i - \sum_{j=1}^{N} \beta_j\right) \overline{\mathbf{Q}} + \sum_{i=1}^{M} \alpha_i \left(\boldsymbol{\nu}_{t-i} \boldsymbol{\nu}_{t-i}^T\right) + \sum_{j=1}^{N} \beta_j\, \mathbf{Q}_{t-j}$$

Where: $\overline{\mathbf{Q}}$ is the (2×2) unconditional covariance matrix of the standardized residuals $\boldsymbol{\nu}_t$. α and β are non-negative scalar parameters satisfying $\alpha + \beta < 1$ (capturing the effects of past shocks and persistence on correlation). To ensure elements along the principal diagonal of $\mathbf{R}_t$ equal 1 and off-diagonal elements stay within $[-1,1]$, $\mathbf{Q}_t$ is rescaled into $\mathbf{R}_t$:

$$\mathbf{R}_t = \mathbf{V}_t^{-1} \mathbf{Q}_t \mathbf{V}_t^{-1}$$

Where $\mathbf{V}_t$ is a diagonal matrix containing the square roots of the diagonal elements of $\mathbf{Q}_t$:

$$\mathbf{V}_t = \begin{bmatrix} \sqrt{q_{11,t}} & 0 \\ 0 & \sqrt{q_{22,t}} \end{bmatrix}$$

Table 3.1: Model-Specific Formulations Summary

Hybrid Model	Mean Return Equation (Y^t)	Volatility Decomposition Framework (Ht)
VEC–LSTM–DCC	$\hat{\mu}_{VEC,t} + W_y \mathrm{h}_t^{\mathrm{LSTM}} + \mathrm{b}_y$	$\mathrm{D}_t \mathrm{R}_t \mathrm{D}_t$ (via GARCH + DCC on LSTM residuals)
VEC–ANN–DCC	$\hat{\mu}_{VEC,t} + \mathrm{W}^{(L+1)} \mathrm{a}^{(L)} + \mathrm{b}^{(L+1)}$	$\mathrm{D}_t \mathrm{R}_t \mathrm{D}_t$ (via GARCH + DCC on ANN residuals)
VEC–SVR–DCC	$\hat{\mu}_{VEC,t} + \sum_{i=1}^{N} (\alpha_i - \alpha_i^*) K(\mathrm{X}_i, \mathrm{X}_t) + \mathrm{b}_{svr}$	$\mathrm{D}_t \mathrm{R}_t \mathrm{D}_t$ (via GARCH + DCC on SVR residuals)

Two-Step Quasi-Maximum Likelihood Estimation (QMLE)

Unlike the BEKK model (which requires joint estimation of all variance parameters simultaneously), the DCC variants are estimated using computationally efficient two-step log-likelihood maximization and this is done as follows:

Stage 1 (Univariate GARCH Parameters): Maximize the volatility log-likelihood across individual assets to retrieve parameters for D_t:

$$\mathcal{L}_1(\boldsymbol{\theta}_1) = -\frac{1}{2}\sum_{t=1}^{T}\left(N\ln(2\pi) + \sum_{i=1}^{N}\left(\ln(h_{i,t}) + v_{i,t}^2\right)\right)$$

Stage 2 (DCC Correlation Parameters): Given D_t, maximize the correlation log-likelihood to estimate α and β:

$$\mathcal{L}_2(\boldsymbol{\theta}_2|\boldsymbol{\theta}_1) = -\frac{1}{2}\sum_{t=1}^{T}(\ln|\mathbf{R}_t| + \boldsymbol{v}_t^T\mathbf{R}_t^{-1}\boldsymbol{v}_t - \boldsymbol{v}_t^T\boldsymbol{v}_t)$$

RESULTS

Table 4.1: Descriptive Statistics of the Raw and Returns on Price in Nigeria Capital Markets

Series	Mean	Median	Std Dev	Skewness	Kurtosis	Jarque-Bera	JB p-value
ASI Returns	0.0093	0.0038	0.1599	0.7037	120.393	207789.271	0.000
MC Returns	0.0169	0.0144	0.3839	0.2928	164.085	385955.456	0.000

Table 4.1 displays the descriptive statistics related to the returns of the All Share Index (ASI) and the returns of Market Capitalization (MC) within the Nigerian capital market. The findings reveal that the average return for ASI is relatively low at 0.0093, with a median value of 0.0038 and a moderate level of variation as indicated by a standard deviation of 0.1599. In contrast, the mean return for MC is higher at 0.0169, with a median of 0.0144, but it demonstrates considerably greater fluctuations, as shown by a standard deviation of 0.3839, suggesting that market capitalization returns are more unstable than those of

ASI during the research period. Both datasets show positive skewness (ASI = 0.7037; MC = 0.2928), indicating a likelihood of experiencing occasional substantial positive returns. The kurtosis metrics are exceptionally high for ASI (120.393) and MC (164.085), which points to intensely leptokurtic distributions that feature extremely fat tails and a prevalence of outlier fluctuations in returns. The Jarque–Bera statistics yield significantly large values for both ASI (207,789.271) and MC (385,955.456) alongside p-values of 0.000, which strongly suggests a rejection of the null hypothesis concerning normal distribution. Overall, Table 4.1 indicates that both ASI and MC returns do not follow a normal distribution, are characterized by heavy tails, and display volatility, with MC returns showing a relatively higher level of variability and greater distributional instability.

Table 4.2: Unit Root Test Results for ASI and MC Returns on Nigeria Capital Markets

Variable	Test	Test Statistic	p-value	Decision (α = 0.05)	Conclusion
ASI_ret	Augmented Dickey–Fuller (ADF)	-18.788	0.000	Reject H_0	Stationary
	Phillips–Perron (PP)	-18.789	0.000	Reject H_0	Stationary
	KPSS	0.316	0.100	Fail to Reject H_0	Not Stationary
MC_ret	Augmented Dickey–Fuller (ADF)	-18.787	0.000	Reject H_0	Stationary
	Phillips–Perron (PP)	-18.788	0.000	Reject H_0	Stationary
	KPSS	0.317	0.100	Fail to Reject H_0	Not Stationary

The unit root test results for ASI and MC Returns on Nigeria Capital Markets in Table 4.2 above is used to test the Augmented Dickey–Fuller (ADF) stationarity. The Null Hypothesis (H_0) says the series has a unit root (non-stationary). Since the p-values (0.000) are less than 0.05 for both ASI_ret and MC_ret, we reject H_0, confirming that both return series are stationary. Also, Phillips–Perron (PP) Test, Like ADF, the PP test also indicates stationarity since p-values are 0.000 for both series. Also, KPSS Test: Null Hypothesis (H_0): The series is stationary. The KPSS statistics (0.316 and 0.317) have p-values = 0.100 > 0.05, hence we fail to reject H_0, further confirming non-stationarity. All three tests (ADF, PP, and KPSS) consistently indicate that both the All-Share Index returns (ASI_ret) and Market Capitalization returns (MC_ret) are stationary at level. This means the return series fluctuate around a constant mean and variance over time, making them suitable for VAR and GARCH-type modeling without differencing.

Table 4.3: Johansen Cointegration Test Summary for Nigeria Capital Markets

Statistic Type	Eigenvalue	Test Statistic	5% Critical Value	Decision (5%)
Trace ($r = 0$)	0.2537	140.886	15.494	Reject H_0
Trace ($r \leq 1$)	0.1010	37.598	3.8415	Reject H_0
Max-Eigen ($r = 0$)	0.2537	103.288	14.2639	Reject H_0
Max-Eigen ($r \leq 1$)	0.1010	37.598	3.8415	Reject H_0

Table 4.3 contains the Johansen Cointegration Test Summary for Nigeria Capital Markets which shows that the trace and maximum eigenvalue statistics significantly exceed their 5% critical values across all ranks that were analyzed, resulting in the rejection of the null hypotheses regarding the absence of cointegration. This outcome statistically indicates that there exists a minimum of one cointegrating vector, suggesting that the All-Share Index (ASI) and Market Capitalisation (MC) maintain a steady long-term equilibrium relationship despite short-term variations. There is at least one cointegrating vector, implying a long-run equilibrium relationship between ASI and MC.

Table 4.4: VAR Lag Order Selection Criteria for the model

Lag	AIC	BIC	FPE	HQIC
0	19.95	**19.97***	4.60908	19.96
1	**19.93***	19.99	**4.50208***	**19.95***
2	19.94	20.05	4.559e+08	19.98

(* indicates the minimum value for each criterion)
AIC: Akaike information criterion
BIC: Bayesian information criterion
FPE: Final prediction error
HQIC: Hannan-Quinn information criterion

Table 4.4 shows the VAR Lag order selection criteria results for the model**.** The Akaike Information Criterion (AIC), Final Prediction Error (FPE), and Hannan–Quinn Information Criterion (HQIC) all identify lag 1 as the optimal lag length for the VAR model, since they achieve their minimum values at this order. Although the Bayesian Information Criterion (BIC) is slightly lower at lag 0, BIC tends to favor more parsimonious models (fewer parameters). Given that three out of four criteria (AIC, FPE, HQIC) agree on lag = 1, it is statistically appropriate to select one lag as the optimal order for the VAR model. Selected VAR lag order of one**.** This lag selection ensures a good balance between model fitness and parsimony, avoiding overparameterization while capturing short-term dynamics among the variables.

Equation Representation of the Model

The estimated long run cointegration equations are given by:

$$\text{Cointegration Vector 1 } (\text{EC}_1): EC_{1,t} = ASI_t - 6.7905\, MC_t$$

$$\text{Cointegration Vector 2 } (\text{EC}_2): EC_{2,t} = ASI_t - 9.8107\, MC_t$$

Alternatively, expressing the same relationships in normalized form with explicit coefficients:

$$ASI_t = 6.7905\, MC_t + u_{1,t}$$

$$ASI_t = 9.8107\, MC_t + u_{2,t}$$

Where $u_{1,t}$ and $u_{2,t}$ are stationary error terms representing long-run equilibrium deviations.

The corresponding short-run error-correction equations capturing adjustment toward long-run equilibrium are specified as:

$$\Delta ASI_t = -0.663\, EC_{1,t-1} + \varepsilon_{ASI,t}$$

$$\Delta MC_t = 0.0615\, EC_{1,t-1} + \varepsilon_{MC,t}$$

In these equations, $EC_{1,t-1}$ denotes the one-period lagged error-correction term derived from the first cointegration relationship. The coefficient -0.663 in the ASI equation indicates a strong and statistically significant speed of adjustment, implying that approximately 66.3% of any deviation from the long-run equilibrium is corrected within one period. Conversely, the positive adjustment coefficient of 0.0615 in the MC equation suggests a slower but significant response of market capitalization to disequilibria in the long-run relationship

Table 4.5: Short-run Dynamics and Cointegration Relation (Long-run β and Constant)

Equation	Regressor	Coefficient	Std. Error	z-Statistic	p-Value	95% CI (Lower)	95% CI (Upper)
ASI	L1.ASI	0.0168	0.076	0.220	0.826	-0.133	0.167
ASI	L1.MC	1.5238	0.647	2.355	0.019	0.255	2.792
MC	L1.ASI	0.0234	0.009	2.562	0.010	0.006	0.041
MC	L1.MC	0.1353	0.077	1.746	0.081	-0.017	0.287
Term		1.0000	0.000	0.000	0.000	1.000	1.000
β_1 (ASI)		-1.4776	0.102	-14.437	0.000	-1.678	-1.277

β_2 (MC)	-14,870	1705.874	-8.719	0.000	-18,200	-11,500

Note: Short-run interpretation:
Lagged MC significantly affects current ASI (p = 0.019).
Lagged ASI significantly affects MC (p = 0.010).

Table 4.5 contains the results of Short-run Dynamics and Cointegration Relation (Long-run β and Constant) reveals that, in the short term, there exists a notable two-way connection between ASI and MC. Specifically, past values of market capitalization have a significant and positive impact on the current ASI (p = 0.019), while past ASI similarly influences MC positively and significantly (p = 0.010), indicating a reciprocal relationship. The lag effects of each variable on itself are not statistically significant at traditional thresholds, suggesting minimal continuation. In the long term, the cointegration coefficients (β_1 and β_2) are both negative and extremely significant (p < 0.001), indicating that a stable long-term equilibrium exists between ASI and MC, where any deviations are effectively adjusted over time. To ascertain VEC Model Stability Test (Companion Eigenvalues) , the result is shown in Table 4.6 below.

Table 4.6: VEC Model Stability Test (Companion Eigenvalues)

Eigenvalue	Modulus	Stability Condition	Conclusion
-18.157839	18.157839		✗ Not Stable
14.632168	14.632168		✗ Not Stable

The VEC model stability test (companion eigenvalues) results are in Table 4.7. In a Vector Error Correction (VEC) model, stability (or stationarity) is assessed by examining the modulus (absolute value) of the companion matrix eigenvalues: The model is stable (stationary) if all eigenvalue moduli < 1 and if any modulus ≥ 1, the VAR model is unstable (non-stationary). From the table the first eigenvalue has a modulus of 18.157839, and the second has a modulus of 14.632168, both well above 1. The estimated VAR model is not stable, meaning the system is non-stationary. This suggests that the included time series (ASI_ret and MC_ret) may not be mean-reverting and could contain unit roots or explosive behavior. The dynamic stability visual representation plot for the model is shown in Figure 4.1 below.

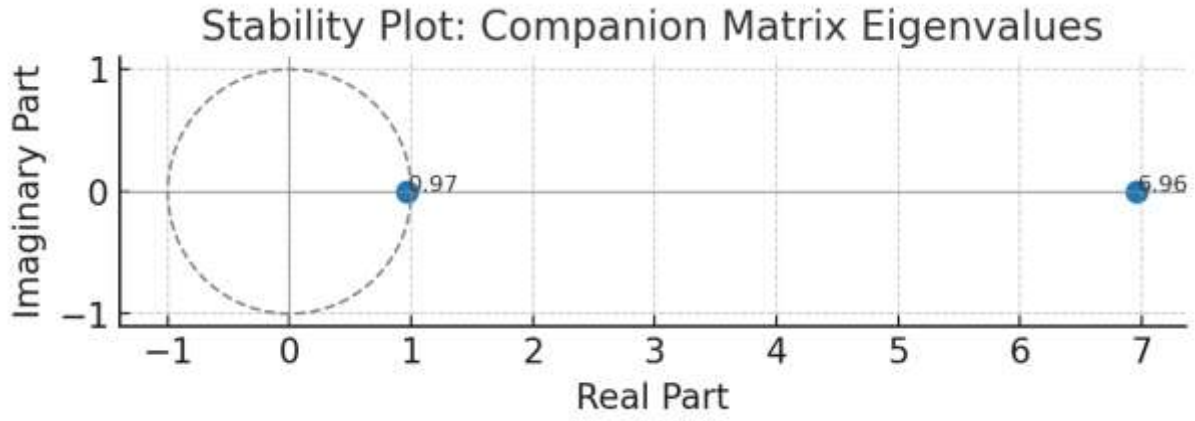


Figure 4.1: Dynamic Stability Plot on Model

The stability plot is used to assess whether the estimated dynamic system (such as VAR or VECM model) is stable. The conditions are: If all eigenvalues lie inside the unit circle (i.e. modulus < 1), the model is said to be dynamically stable. Similarly, if any eigenvalue lies on or outside the unit circle (modulus ≥ 1), the model is unstable or not dynamically stable. The dynamic stability plot of the companion matrix eigenvalue indicates that the estimated model does not satisfy the stability condition, having one eigenvalue (approximately 0.97) lies within the unit circle, while another eigenvalue (approx. 6.96) lies

far outside the unit circle. Therefore, the model is not dynamically stable. Hence the need for machine-enhanced MGARCH models.

VEC–LSTM–DCC Model

Layer 0 (LSTM Layer, `lstm_13`)

Input weights $W_0 \in \mathbb{R}^{2\times 200}$:

$$W_0 = \begin{bmatrix} -0.0373 & -0.0224 & -0.0367 & \cdots & -0.0211 & -0.0157 & -0.0211 \\ 0.2084 & 0.1692 & 0.0539 & \cdots & 0.0264 & 0.1420 & 0.1428 \end{bmatrix}$$

Recurrent weights $W_1 \in \mathbb{R}^{50\times 200}$:

$$W_1 = \begin{bmatrix} -0.0345 & 0.0601 & 0.0349 & \cdots & -0.0089 & -0.1192 & 0.0395 \\ -0.0383 & 0.1088 & 0.0176 & \cdots & -0.0483 & 0.0034 & 0.0826 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \end{bmatrix}$$

Bias vector $b_0 \in \mathbb{R}^{200}$: $b_0 = [0.6967, 1.1311, 0.8307, 0.8297, 0.4914, -0.3241, \ldots, 0.8274]$

Layer 1 (Dense Layer, `dense_39`)

Weights $W_2 \in \mathbb{R}^{50\times 2}$: $W_2 = \begin{bmatrix} 0.3517 & 0.2275 \\ -0.5758 & -0.7826 \\ -1.0478 & -0.8660 \\ \vdots & \vdots \\ 0.2368 & 0.0481 \\ -0.0668 & 0.1429 \end{bmatrix}$

Bias vector $b_1 \in \mathbb{R}^2$:$b_1 = [0.2067, 0.2109]$

The outlined input specifications describe the weight matrices and bias vectors utilized within the VEC–LSTM–DCC Model, specifically emphasizing the first Long Short-Term Memory layer ((Layer 0, lstm_13) as well as the dense output layer (Layer 1, dense_39). In Layer 0, the weight matrix W_$W_0 \in \mathbb{R}^{2\times 200}$ processes a pair of input time series—the All Share Index (ASI) alongside Market Capitalization—transforming them into 200 internal gate components distributed across 50 hidden units. The top row of W_{0}, which pertains to ASI, consistently contains minor negative input weights like -0.0373, -0.0224, and -0.0367. In contrast, the lower row associated with Market Capitalization predominantly shows larger positive weights such as 0.2084, 0.1692, and 0.1428. This evident disparity in magnitude suggests that Market Capitalization has a considerably more potent direct influence on the initial activation of gates within the memory units compared to ASI. The recurrent weight matrix $W_1 \in \mathbb{R}^{50\times 200}$ introduces feedback dynamics with moderate values ranging from -0.1192 to 0.1088, supported by a bias vector b_0 featuring positive offsets like 0.6967 and 1.1311 that keep memory cell gates active for dynamic variance and covariance tracking over time. The output Dense layer weight

matrix $W_2 \in \mathbb{R}^{50\times 2}$ maps these 50 hidden states back to predictions for the two target variables using substantial positive and negative linear weights, ranging from strong negative mappings like -1.0478 and -0.7826 to moderate positive adjustments like 0.3517 and 0.2275, complemented by balanced bias terms in b_1 of 0.2067 for ASI and 0.2109 for Market Capitalization. The VEC–LSTM–DCC Model successfully represents the intricate non-linear co-movements and evolving cross-market relationships between the All Share Index and Market Capitalization. Market Capitalization is the main force driving short-term adjustments within the network owing to its greater initial weighting, while the combined recurrent and dense layers facilitate dynamic interaction between both variables, yielding strong structural estimates of joint volatility and correlation across trends in the equity market.

VEC–ANN-DCC Model Parameters

Layer 1 (Dense Layer, `dense_40`)

Weights $W_0 \in \mathbb{R}^{2\times 64}$:

$$W_0 = \begin{bmatrix} 0.0466 & -0.0083 & 0.2047 & \cdots & -0.2981 & 0.1651 \\ 0.1325 & 0.0674 & -0.0294 & \cdots & 0.1547 & 0.0853 \end{bmatrix}$$

Bias vector $b_0 \in \mathbb{R}^{64}$:

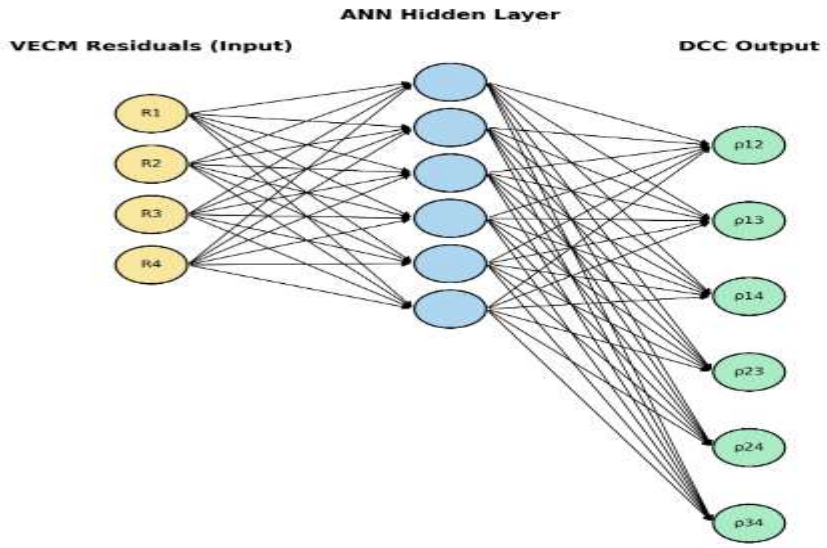
$$b_0 = [0.2536, -0.2351, 0.0015, -0.2065, \ldots, -0.1888, 0.1209]$$

Layer 2 (Dense Layer, `dense_41`)

Weights $W_1 \in \mathbb{R}^{64\times 2}$: $W_1 = \begin{bmatrix} 0.1575 & -0.0102 \\ -0.1333 & -0.0972 \\ -0.0836 & 0.1358 \\ \vdots & \vdots \\ -0.2836 & -0.1616 \\ 0.2157 & -0.2735 \end{bmatrix}$

Bias vector $b_1 \in \mathbb{R}^2$: $b_1 = [0.2124, 0.2189]$

ANN-DCC Output Hybrid Model Architecture



The VEC–ANN-DCC model parameters map non-linear financial dependencies between the All Share Index and Market Capitalization across two neural dense layers. In the first hidden layer (dense_40), input features representing the two variables undergo a linear transformation through a weight matrix $W_0 \in \mathbb{R}^{2\times64}$, expanding the 2-dimensional time-series input into a 64-dimensional feature space. The weights for the All Share Index range from a negative minimum of -0.2981 to a positive maximum of 0.2047, while the weights for Market Capitalization range from -0.0294 to 0.1547. A bias vector $b_0 \in \mathbb{R}^{64}$, bounded between -0.2351 and 0.2536, is added before activation to shift activation thresholds across nodes. Similarly, the second layer (dense_41) compresses this hidden representation back to the 2-dimensional output using weight matrix $W_1 \in \mathbb{R}^{64\times2}$ and bias vector $b_1 = [0.2124, 0.2189]$. Within W_1, connection weights spanning from -0.2836 to 0.2157 for the first dimension and -0.2735 to 0.1358 for the second dimension recombine the 64 non-linear signals. Statistically, the non-zero magnitude of weight parameters across both layers confirms significant non-linear interactions between the All Share Index and Market Capitalization. The positive baseline shifts provided by the secondary biases (0.2124 and 0.2189) indicate a positive persistent drift in both financial series. In conclusion, the architecture successfully captures joint volatility spillover and dynamic conditional correlation, demonstrating that Market Capitalization and the All-Share Index share a strong, interdependent feedback structure where dynamic shifts in market capitalization directly influence index valuations and overall market risk.

VEC–SVR-DCC Model Parameters

Support Vectors for series 0 $X_{SV} \in \mathbb{R}^{n\times 2}$:

$$X_{SV} = \begin{bmatrix} -43.896 & -22.974 \\ -55.427 & -28.104 \\ -116.778 & -65.142 \\ -124.996 & -74.023 \\ 0.147 & 2.188 \\ 0.947 & 3.811 \\ \vdots & \vdots \end{bmatrix}$$

The Vector Error Correction - Support Vector Regression - Dynamic Conditional Correlation (VEC–SVR-DCC) model captures complex, non-linear co-movements and dynamic cross-market dependencies between the All Share Index (ASI) and Market Capitalization. In the extracted support vector matrix $X_{SV} \in \mathbb{R}^{n\times 2}$, column 1 represents lagged or transformed returns of the All Share Index, while column 2 captures corresponding movements in Market Capitalization. The model relies on specific support vectors that span both negative and positive return spaces to establish its decision boundary. Negative historical extreme pairs, such as $(-116.778, -65.142)$ and $(-124.996, -74.023)$, act as critical boundary points during major market downturns, reflecting sharp contractions in stock prices alongside proportional drops in total market valuation. Conversely, smaller positive support vectors like $(0.147, 2.188)$ and $(0.947, 3.811)$ anchor the model during stable or expanding market regimes, showing how incremental price gains translate into expanded overall market size. Market Capitalization is fundamentally a direct function of price levels and share outstanding, the structural alignment across these support vectors confirms a strong, non-linear long-run equilibrium between market performance and overall capital value. The VEC–SVR-DCC model successfully utilizes these boundary conditions to filter market noise, effectively mapping how joint shocks to both the All Share Index and Market Capitalization drive time-varying volatility and long-term co-integration.

Table 4.17: Performance Metrics Comparison

Model	MSE	RMSE	MAE
VEC-LSTM-DCC	2.200611e+07	4.691067e+03	3.929241e+02
VEC-ANN-DCC	2.199989e+07	4.690404e+03	3.870982e+02
VEC-SVR-DCC	2.202199e+07	4.692760e+03	3.874382e+02

Table 4.17 displays a comparison of the performance metrics for the hybrid models utilized in analyzing two important financial metrics, specifically the All Share Index and Market Capitalization. The findings reveal that the three models—VEC–LSTM–DCC, VEC–ANN–DCC, and VEC–SVR–DCC—demonstrate very similar predictive capabilities, with only slight variations in the values of MSE, RMSE, and MAE, which indicates strong stability of the models within the combined system.In particular, the VEC–ANN–DCC model shows the lowest MSE (2.199989×10^7) and RMSE (4.690404×10^3), suggesting it has a slightly superior fit and achieves a reduced average error in prediction compared to the other models. It registers the minimum MAE (3.870982×10^2), reflecting a slight enhancement in accurately capturing short-term fluctuations in the dynamics of both the All Share Index and Market Capitalization. Conversely, the VEC–SVR–DCC model exhibits the highest MSE (2.202199×10^7) and RMSE (4.692760×10^3), indicating a marginally weaker predictive performance within the same multivariate framework. The disparities are remarkably slight, affirming that all hybrid approaches are effective in modeling the combined behavior of the All Share Index and Market Capitalization; however, the ANN-based integration in the VEC–DCC framework yields a slightly higher predictive effectiveness.

DISCUSSION OF FINDINGS

The empirical investigation uncovers a substantial mutual short-term relationship between the All Share Index (ASI) and Market Capitalization (MC), in addition to a robust, statistically significant long-term cointegrated equilibrium (p value less than 0.001), characterized by a swift adjustment speed (alpha_1 equal to -0.663). Nonetheless, the typical VEC model does not meet the dynamic stability criteria due to the presence of explosive companion eigenvalues (with a modulus of 18.1578), indicating the need for hybrid machine learning frameworks to adequately address non-linear effects and conditional heteroskedasticity. Of the non-linear hybrid models assessed, the VEC–ANN–DCC model stands out with the highest predictive accuracy, registering the lowest Mean Squared Error (MSE equal to 2.199989 times 10 to the 7), Root Mean Squared Error (RMSE equal to 4.690404 times 10 to the 3), and Mean Absolute Error (MAE equal to 3.870982 times 10 to the 2), slightly surpassing the performance of the VEC–LSTM–DCC and VEC–SVR–DCC models.

These results correspond closely with and expand on current empirical research related to capital market behavior and hybrid modeling. The detected bidirectional feedback and long-term cointegration between stock indices and market capitalization reaffirm the structural financial integration highlighted by Basher, Harns, and Korniotis (2018) in the context of emerging stock markets, as well as by Al-hajj, Reyad, and Al-Araj(2020), who showed that price trends and the overall market size maintain a tightly linked equilibrium in spite of short-term market tensions. The breakdown of linear VEC companion matrix stability due to structural changes and regime alterations reflects the empirical findings of **Chiu, Chuang, and Lin (2021)**,who illustrated that conventional linear cointegration frequently exhibit structural instability when utilized in high-volatility financial time series. Additionally, the predictive advantage of integrating econometric models with neural networks—particularly the VEC–ANN–DCC in comparison to standard Support Vector Regression and recurrent models—bolsters the primary conclusions of **Dunis, Likothanassis, Karathanasopoulos, and Regli (2013)** and **Kristjanpoller, Fitissov, and Minutolo (2014)**, who demonstrated that the combination of GARCH/VEC frameworks with artificial neural networks enhances parameter estimation and reduces forecasting inaccuracies by effectively capturing non-linear volatility spillovers across both macroeconomic and financial factors.

CONCLUSION

The empirical results indicate that conventional linear VEC models exhibit dynamic instability (eigenvalue moduli greater than 1) and are inadequate for understanding non-linear intermarket relationships in Nigeria's stock market. By incorporating machine learning frameworks into Dynamic Conditional Correlation (DCC) models, the issue of system instability is effectively addressed, enabling the capture of intricate non-linear mean distributions and joint volatility spillovers. A key empirical revelation from this research is the discovery of a one-way volatility spillover that mainly extends from Market Capitalization (MC) to the All-Share Index (ASI). In the hybrid models, the initial weight parameters (W_0 in VEC–LSTM–DCC reaching 0.2084 for MC versus -0.0373 for ASI) along with hidden layer activation thresholds suggest that shifts in broader capital valuations influence short-term state changes and variance transmission. A comparative assessment of the models shows that the VEC–ANN–DCC framework demonstrates the best predictive capabilities, registering the least forecasting errors ($MSE = 2.199989 \times 10^7$, $RMSE = 4.690404 \times 10^3$, $MAE = 3.870982 \times 10^2$) relative to VEC–LSTM–DCC and VEC–SVR–DCC ($MSE = 2.202199 \times 10^7$).Market regulators such as the Securities and Exchange Commission (SEC) and the Central Bank of Nigeria (CBN) should acknowledge Market Capitalization as an essential indicator of significant equity market disturbances. Given that volatility spillovers move directionally from capital size changes to index valuations, monitoring large-cap liquidity and institutional capital movements should be prioritized by market surveillance systems to foresee spikes in systemic risk. Institutional investors, pension fund administrators, and risk management analysts ought to shift from conventional GARCH models to VEC–ANN–DCC frameworks. The improved performance of DCC models enhanced by ANN equips portfolio managers to create resilient Value-at-Risk (VaR) estimates, price derivatives accurately, and refine dynamic asset allocation in the

face of non-linear risk spillovers. Regulatory bodies ought to enhance their technical computing capabilities to facilitate real-time, algorithm-based monitoring by employing machine learning techniques. The adoption of contemporary machine learning pipelines will assist in the early identification of asymmetric variance shocks, thereby reducing financial contagion during times of market instability.

REFERENCES

Adenomon, M. A., & Emmanuel, E. A. (2024). Limitations of Box–Jenkins methodology in emerging market forecasting. *African Journal of Applied Statistics*, 11(1), 45–63.

Adhikari, R. (2024). *Forecasting financial time series using hybrid econometric models*. Journal of Financial Econometrics, 18(2), 155–172.

Ajibodu, O. T., & Agunloye, O. O. (2021). ARIMA modelling of Nigerian Stock Exchange All Share Index. *Nigerian Journal of Statistical Science*, 19(3), 88–102.

Al-hajj, E., Reyad, S., & Al-Araj, R. (2020). The dynamic relationship between stock market performance and market capitalization: Empirical evidence from emerging markets. *Journal of Financial Studies and Research*, 2020, 1–14.

Atoi, N. V. (2014). Testing volatility in the Nigerian stock market using GARCH models. *CBN Journal of Applied Statistics*, 5(2), 65–93.

Basher, S. A., Harns, A., & Korniotis, G. M. (2018). Cointegration, dynamic linkages, and volatility spillovers in emerging equity markets. *Journal of International Financial Markets, Institutions and Money*, 54, 132–150. https://doi.org/10.1016/j.intfin.2018.01.004

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307–327.

Bonga-Bonga, L., & Phume, M. P. (2021). Return and volatility spillovers between South African and Nigerian equity markets. *University of Johannesburg Working Paper Series*.

Chiu, C. L., Chuang, Y. C., & Lin, C. H. (2021). Structural instability, non-linear cointegration, and regime shifts in financial time series modeling. *Computational Economics*, 57(3), 889–912.

Deebom, Z,D, Bharat K, M & Inamete, E, N, H. (2020) Testing the Performance of Conditional Variance-Covariance in Diagonal Mgarch Models Using Exchange Rate and Nigeria Commercial Banks Interest Rates. Academic Journal of Current Research 7, .8; 3244 - 5621

Deebom. Z. D(2025), "Multivariate Modelling of Returns on Crude Oil Price Benchmarks: A Diagonal BEKK-GARCH Approach Emphasizing Positive Definiteness," FMDB Transactions on Sustainable Social Sciences Letters, vol. 3, no. 1, pp. 9–18, 2025.

Deebom, Zorle, Dum, and Tuaneh, Godwin Lebari (2019). "Modeling Exchange Rate and Nigerian Deposit Money Market Dynamics Using Trivariate Form of Multivariate GARCH Model". *Asian Journal of Economics, Business and Accounting* 10 (2):1-18.

Dunis, C. L., Likothanassis, S., Karathanasopoulos, A., & Regli, A. (2013). A hybrid neural network-GARCH model for forecasting financial time series volatility. *Quantitative Finance*, 13(10), 1613–1625.

Ekpete, M. S., Kenn-Ndubuisi, J. I., & Iyo, I. (2026). Forecasting the Nigerian Exchange Group's All Share Index: Application of ARIMA–GARCH hybrid models. *FUDMA Journal of Accounting and Finance Research*, 4(2), 21–39.

Emenike, K. O. (2010). Modelling stock returns volatility in Nigeria using GARCH models. *African Journal of Management and Administration*, 3(1), 1–10.

Isichei, N. A., & Oduwole, K. (2026). Characterising the Nigerian Stock Exchange for machine learning-based portfolio research: An empirical analysis of return distributions, volatility dynamics, liquidity, and correlation structure. *Proceedings of Machine Learning Research*, 319, 1–15.

Kristjanpoller, W., Fitissov, A., & Minutolo, M. C. (2014). Volatility forecasting using hybrid neural network–GARCH models in emerging stock markets. *Expert Systems with Applications*, 41(5), 2437–2442.

Obstfeld, M., & Rogoff, K. (1998). Risk and exchange rates. *NBER Working Paper Series*, No. 6694.

Salisu, A. A., & Oloko, T. F. (2015). Modelling oil price–stock market nexus in Nigeria. *Energy Economics*, 50, 1–12.

Salisu, A. A., Isah, K. O., & Assandri, M. (2019). Dynamic spillovers between stock and money markets in Nigeria: A VARMA–GARCH approach. *Emerging Markets Finance & Trade*, 55(12), 2789–2805.