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Feature Sensitivity And Impact Analysis Of Foreign Trade Indicators On GDP Forecasting Using A Hybrid Lagrange–LSTM Framework

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ABSTRACT
Accurate GDP forecasting in developing economies requires both predictive accuracy and an understanding of the variables driving forecast performance. This study investigates the sensitivity and impact of foreign trade indicators on Nigeria's GDP forecasting using a Hybrid Lagrange Interpolation–Long Short-Term Memory (LIF–LSTM) framework. The annual study series covers 2011–2024 and comprises exports, imports, foreign direct investment (FDI), exchange rate, foreign reserves, and GDP. Lagrange interpolation is used as a preprocessing layer to improve continuity and reduce irregularities before LSTM sequence learning. A permutation-based sensitivity analysis is then used to assess the relative contribution of each predictor by examining the deterioration in predictive performance after feature perturbation. The LIF–LSTM model achieved RMSE = 13.5, MAE = 11.2, MAPE = 2.4%, and R^2 = 0.985 on the 2022–2024 test period, outperforming LIF–GRU, ARIMA, and Multiple Linear Regression. The sensitivity results identify exchange rate as the most influential predictor, followed by exports, while imports show the lowest reported contribution. The results indicate that combining mathematical preprocessing, recurrent learning, and feature-importance analysis can support both accurate forecasting and more interpretable macroeconomic decision-making.
Keywords: Explainable AI; Feature importance; Foreign trade; GDP forecasting; LSTM.

1. INTRODUCTION

Gross Domestic Product (GDP) forecasting is important for macroeconomic planning, fiscal management, investment decisions, and the evaluation of economic policy. In Nigeria, GDP dynamics are closely connected with external-sector conditions because exports, imports, foreign investment, exchange-rate movements, and foreign reserves influence production, trade balances, investment incentives, and foreign-exchange availability. The forecasting problem is therefore characterized by interacting variables, nonlinear effects, and structural changes that can be difficult to represent with simple linear specifications (Box et al., 2015), (Fischer & Krauss, 2018).

Traditional models such as Autoregressive Integrated Moving Average (ARIMA) and Multiple Linear Regression (MLR) remain useful because of their established statistical foundations and interpretability. However, recurrent neural networks (RNNs), particularly Long Short-Term Memory (LSTM) and Gated Recurrent Unit (GRU) architectures, provide mechanisms for learning nonlinear temporal dependencies in sequential observations (Hochreiter & Schmidhuber, 1997), (Chung et al., 2014), (Goodfellow et al., 2016). LSTM networks were specifically developed to mitigate limitations associated with learning long-term dependencies, while GRU provides a comparatively compact gated architecture.

A further challenge is data quality. When macroeconomic series contain irregularities, missing observations, or strong local changes, the learning process of a recurrent model may be affected. Lagrange interpolation provides a deterministic numerical mechanism for estimating values from discrete observations and can therefore be used as a preprocessing layer before sequence learning. The present study follows the hybrid modelling direction developed in the accompanying thesis, in which Lagrange interpolation is combined with recurrent learning to improve GDP forecasting from foreign-trade indicators.

Although predictive performance is important, a high-performing model does not by itself explain which variables matter most. Permutation-based variable-importance methods quantify the deterioration in predictive performance when information in a selected feature is disrupted (Fisher et al., 2019). This is particularly relevant to economic forecasting because policy users need information about influential variables in addition to forecast values. Recent AMCI publications also demonstrate the relevance of neural-network forecasting and hybrid recurrent architectures to computational-intelligence applications (Ferdiansyah et al., 2023), (Azizan et al., 2025).

The objective of this study is therefore to examine the sensitivity and impact of foreign-trade indicators on Nigerian GDP forecasting within a Hybrid LIF–LSTM framework. Specifically, the study (i) applies Lagrange interpolation to improve time-series continuity, (ii) evaluates the LIF–LSTM model against LIF–GRU, ARIMA, and MLR, (iii) evaluates predictor sensitivity using permutation-based analysis, and (iv) identifies the most influential trade indicators affecting GDP prediction.

2. MATERIALS AND METHODS

2.1 Data Description

The study uses the annual macroeconomic series documented in the thesis for 2011–2024. The variables are GDP, exports, imports, FDI, exchange rate (NGN/USD), and foreign reserves. Monetary quantities are expressed in USD billions, while the exchange rate is expressed in naira per US dollar. The thesis documents the data sources as the Central Bank of Nigeria (CBN), National Bureau of Statistics (NBS), and World Bank Open Data (Central Bank of Nigeria, 2024; National Bureau of Statistics, 2024; World Bank, 2024).

Table 1. Data sources and study variables.

Source	Indicators used	Period
Central Bank of Nigeria	Exchange rate; GDP and macroeconomic indicators	2011–2024
National Bureau of Statistics	Exports; imports; FDI; GDP	2011–2024
World Bank Open Data	GDP and complementary macroeconomic indicators	2011–2024

The modelling dataset is organized chronologically. Observations from 2011–2021 constitute the training period and observations from 2022–2024 constitute the out-of-sample test period. This chronological split prevents future observations from entering model training and preserves the time-order structure required for forecasting.

2.2 Lagrange Interpolation Preprocessing

For a set of n+1 known observations (x_0,y_0), …, (x_n,y_n), the Lagrange interpolating polynomial is written as:

$$P_n(x) = \Sigma_{i=0}^{n} y_i L_i(x) \quad (1)$$

where the Lagrange basis polynomial is:

$$L_i(x) = \Pi_{j=0,j\neq i}^{n} (x - x_j)/(x_i - x_j) \quad (2)$$

The interpolation layer is applied before neural-network training to reconstruct the working time series and reduce the effect of irregular observations. The interpolated predictors are then transformed to a common scale using Min–Max normalization:

$$z = (x - x_{min})/(x_{max} - x_{min}) \quad (3)$$

The resulting normalized sequences are used as inputs to the recurrent model.

2.3 Hybrid LIF–LSTM Architecture

The proposed architecture contains a Lagrange interpolation preprocessing module followed by an LSTM sequence-learning module and a dense GDP output layer. The LSTM component is designed to retain relevant information across time steps through its gated memory structure (Hochreiter & Schmidhuber, 1997). The thesis architecture uses the foreign-trade indicators as model inputs and GDP as the response variable.

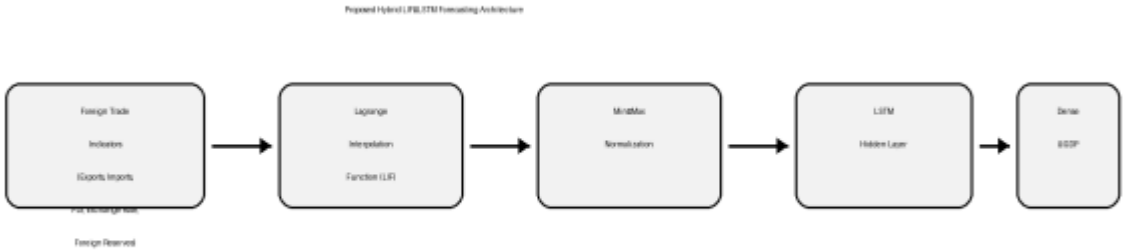

Figure 1. Hybrid LIF–LSTM architecture used for GDP forecasting. Source: Author's thesis framework.

For an LSTM cell, the principal gates can be represented as:

$$f_t = \sigma(W_f x_t + U_f h_{t-1} + b_f) \quad (4)$$

$$i_t = \sigma(W_i x_t + U_i h_{t-1} + b_i) \quad (5)$$

$$o_t = \sigma(W_o x_t + U_o h_{t-1} + b_o) \quad (6)$$

$$h_t = o_t \odot \tanh(c_t) \quad (7)$$

where f_t, i_t, and o_t denote the forget, input, and output gates; h_t is the hidden state; c_t is the cell state; σ is the logistic activation function; and $\odot$ denotes element-wise multiplication.

2.4 Permutation-Based Feature Sensitivity

Permutation sensitivity measures the contribution of an input variable by disrupting that variable while leaving the remaining predictors unchanged. For predictor j, the feature values are randomly permuted over the test observations and the resulting model error is compared with the baseline error. This approach follows the general principle that a feature is important when disrupting its information produces a meaningful deterioration in predictive performance (Fisher et al., 2019).

$$PI_j = (1/R) \Sigma_{r=1}^{R} [E_perm,j,r - E_base] \quad (8)$$

where PI_j is the permutation importance of feature j, R is the number of permutations, E_perm,j,r is the prediction error after permutation r of feature j, and E_base is the baseline prediction error. Larger positive values indicate stronger dependence of model performance on the corresponding feature.

2.5 Evaluation Metrics

Four measures are used to evaluate forecasting performance. For n observations with actual values y_i and predictions $\hat{y}_i$, RMSE, MAE, MAPE, and R^2 are defined as follows:

$$RMSE = \sqrt{[(1/n)\ \Sigma_{i=1}^{n}\ (y_i - \hat{y}_i)^2]} \quad (9)$$

$$MAE = (1/n)\ \Sigma_{i=1}^{n}\ |y_i - \hat{y}_i| \quad (10)$$

$$MAPE = (100/n)\ \Sigma_{i=1}^{n}\ |(y_i - \hat{y}_i)/y_i| \quad (11)$$

$$R^2 = 1 - [\Sigma_i\ (y_i - \hat{y}_i)^2\ /\ \Sigma_i\ (y_i - \bar{y})^2] \quad (12)$$

Lower RMSE, MAE, and MAPE indicate smaller prediction errors, whereas an R^2 value closer to one indicates stronger agreement between observed and predicted values.

3. RESULTS

3.1 Forecasting Model Performance

The final model-comparison results reported in the thesis are reproduced in Table 2. The LIF–LSTM model gives the lowest RMSE, MAE, and MAPE and the highest R^2 among the four evaluated approaches. Its RMSE of 13.5 is lower than 14.2 for LIF–GRU, 18.3 for MLR, and 21.7 for ARIMA. The corresponding R^2 of 0.985 is also the highest.

Table 2. Forecasting performance on the 2022–2024 test period.

Model	RMSE	MAE	MAPE (%)	R^2
LIF–LSTM	13.5	11.2	2.4	0.985
LIF–GRU	14.2	12.0	2.7	0.978
ARIMA	21.7	19.4	4.5	0.926
Multiple Linear Regression	18.3	16.1	3.8	0.948

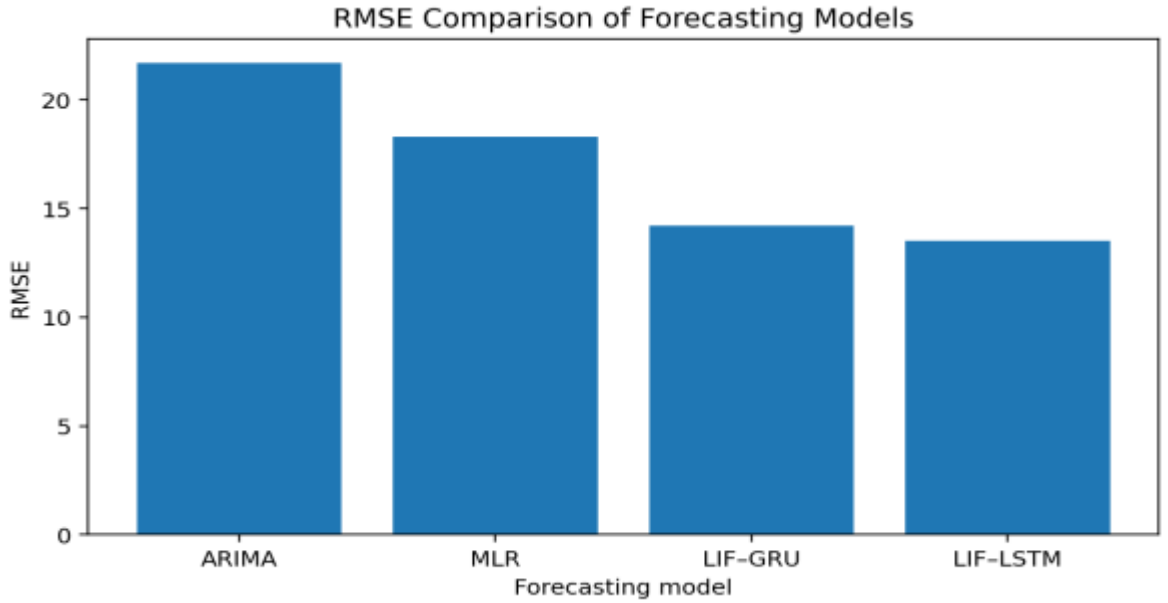

Figure 2. RMSE comparison of the four forecasting approaches. Source: Author's thesis results.

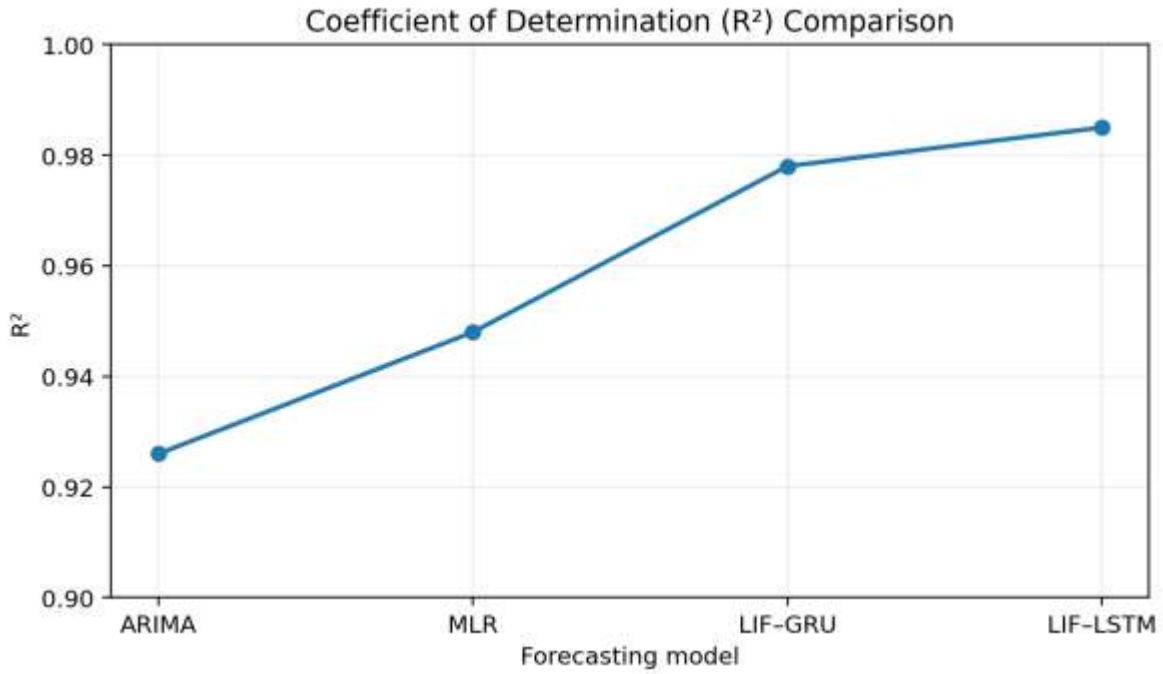

Figure 3. R^2 comparison of the four forecasting approaches. Source: Author's thesis results.

The graphical results reinforce the numerical comparison. The hybrid recurrent models occupy the lower-error region, with LIF–LSTM providing the best overall balance between low prediction error and high explanatory power. The result supports the thesis conclusion that mathematical preprocessing can complement recurrent learning when the underlying economic series are nonlinear and potentially irregular.

3.2 Feature Sensitivity Results

The permutation-based analysis identifies exchange rate as the most influential predictor of GDP, followed by exports. Foreign reserves and FDI are classified as having moderate influence, while imports have the lowest reported contribution. This ordering indicates that the predictive performance of the LIF–LSTM model is particularly sensitive to information associated with exchange-rate movements and export activity.

Table 3. Reported feature sensitivity ranking.

Rank	Variable	Reported impact level
1	Exchange Rate	Very High
2	Exports	High
3	Foreign Reserves	Moderate
4	FDI	Moderate
5	Imports	Low

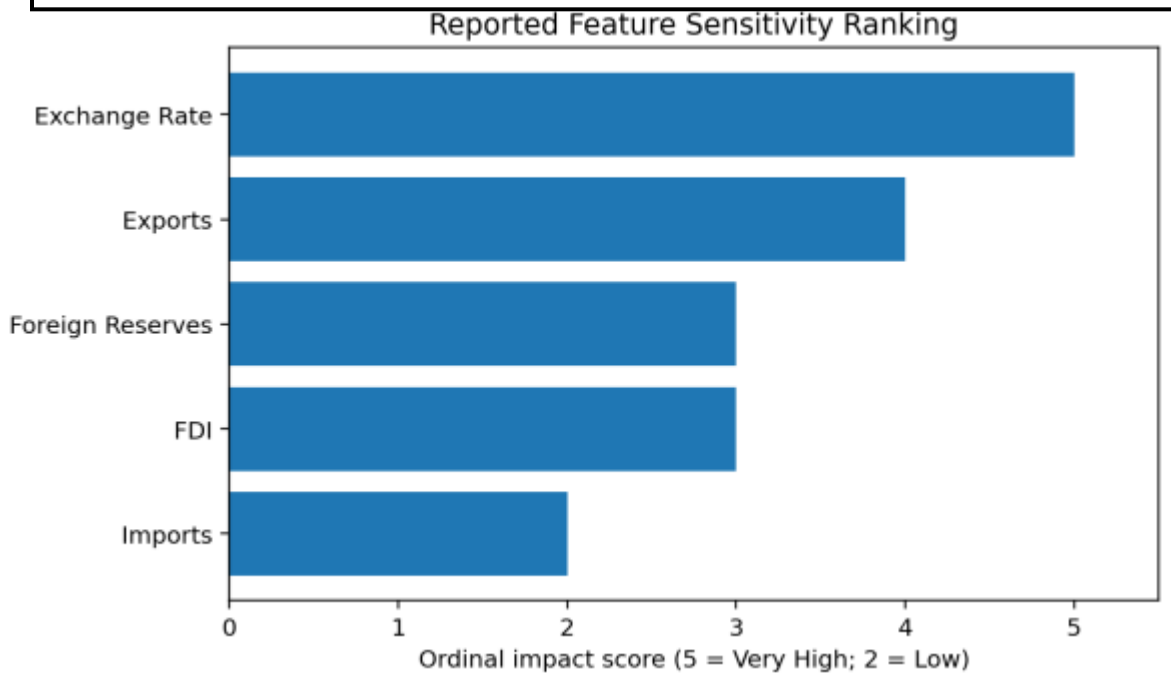

Figure 4. Ordinal visualization of the reported feature-impact categories (5 = Very High; 4 = High; 3 = Moderate; 2 = Low). The scores are visualization codes, not raw permutation-importance values.

Because the manuscript reports the sensitivity results categorically rather than as raw permutation-error increases, Figure 4 uses an ordinal encoding only to make the reported ranking visually interpretable. It should therefore not be interpreted as a quantitative estimate of the underlying permutation-importance magnitudes.

3.3 Actual and Predicted GDP During the Test Period

The thesis graphical diagnostics show close agreement between actual and model-predicted GDP during the held-out period. For the 2022–2024 window, the actual GDP values documented in the thesis are USD 561.0, 576.0, and 590.0 billion. The corresponding predicted values reconstructed from the thesis test-period plot are approximately USD 558.0, 579.0, and 593.0 billion.

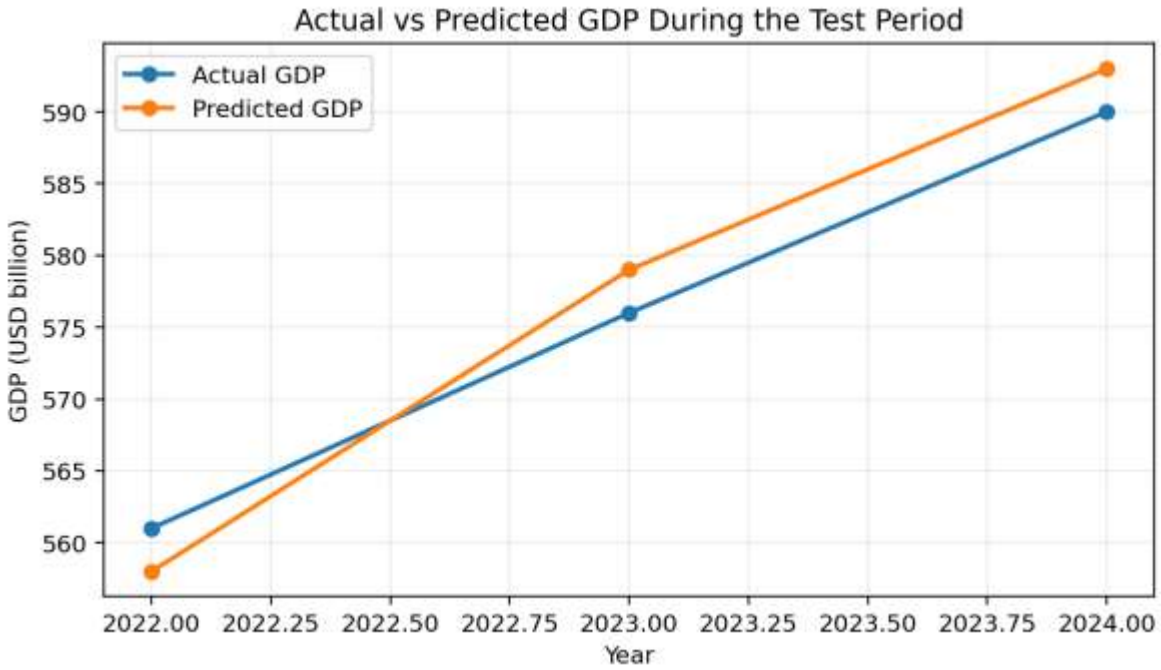

Figure 5. Actual versus predicted GDP for the 2022–2024 test period, reconstructed from the thesis graphical results

3.4 Long-Term GDP Projection Using Lagrange Extrapolation

As an additional application of the numerical component, the thesis uses Lagrange extrapolation to extend the GDP series beyond the observed 2011–2024 period. The resulting projections increase from USD 606.4 billion in 2025 to USD 699.7 billion in 2030.

Table 4. GDP projections from the thesis using Lagrange extrapolation.

Year	Forecast GDP (USD billion)
2025	606.4
2026	623.8
2027	641.9
2028	660.6
2029	679.8
2030	699.7

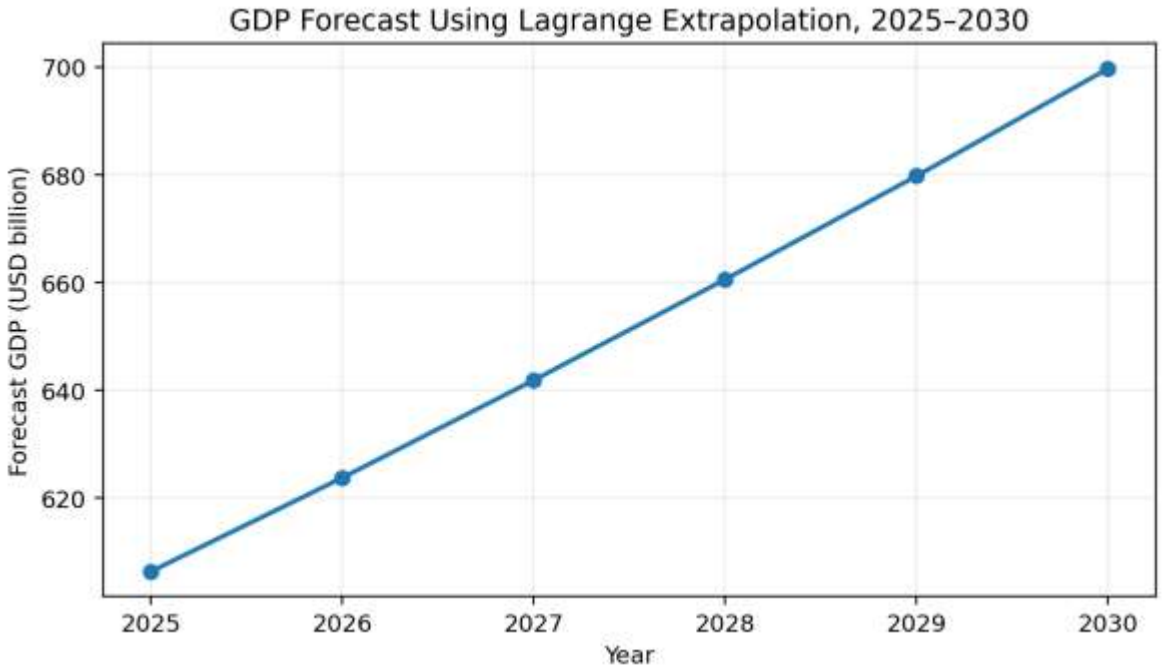

Figure 6. GDP forecast using Lagrange extrapolation, 2025–2030. Source: Author's thesis results

DISCUSSION

The results provide two complementary forms of evidence. First, the LIF–LSTM model achieved the strongest predictive performance among the evaluated methods, with RMSE = 13.5, MAE = 11.2, MAPE = 2.4%, and R^2 = 0.985. The lower errors relative to ARIMA and MLR suggest that the nonlinear sequence-learning capacity of LSTM is useful for the relationships represented in the foreign-trade/GDP series. The comparison with LIF–GRU also indicates that the LSTM configuration used in the study provided a modest advantage over the alternative gated architecture.

Second, the sensitivity analysis adds interpretability to the predictive result. Exchange rate is classified as very highly influential and exports as highly influential. This pattern is consistent with the role of foreign-exchange conditions and export receipts in an economy strongly exposed to international trade. A change in the exchange rate can affect import costs, export competitiveness, investment decisions, and the domestic value of foreign-currency transactions. Similarly, export performance can influence foreign-exchange availability and aggregate economic activity. The results therefore suggest that forecast monitoring should pay particular attention to these variables.

The lower reported importance of imports should be interpreted cautiously. A low permutation contribution within this particular model does not mean that imports are economically unimportant; rather, it means that disrupting the import feature produced a smaller deterioration in the model's predictive performance than disrupting the higher-ranked features. This distinction is important when interpreting machine-learning feature importance in economic applications (Fisher et al., 2019).

The results also demonstrate the value of combining model accuracy with interpretability. Recent AMCI studies have reported the usefulness of neural-network and hybrid architectures for forecasting applications (Ferdiansyah et al., 2023), (Azizan et al., 2025). The present study extends this computational-intelligence perspective to Nigerian macroeconomic forecasting by adding a numerical interpolation stage and an explicit feature-sensitivity analysis.

The 2025–2030 Lagrange projections provide a supplementary long-horizon scenario rather than a substitute for a full probabilistic macroeconomic forecast. Because extrapolation extends beyond the

observed sample, the projected values should be interpreted with caution, particularly under structural changes, commodity-price shocks, exchange-rate reforms, or other unforeseen macroeconomic disturbances.

5. CONCLUSION

This study examined the sensitivity and impact of foreign-trade indicators on Nigerian GDP forecasting using a Hybrid Lagrange Interpolation–LSTM framework. The results show that the LIF–LSTM model achieved the best reported forecasting performance, with RMSE = 13.5, MAE = 11.2, MAPE = 2.4%, and R^2 = 0.985 on the 2022–2024 test period. The model outperformed LIF–GRU, ARIMA, and Multiple Linear Regression on the reported evaluation measures.

The feature-sensitivity analysis further indicates that exchange rate and exports are the most influential predictors, followed by foreign reserves and FDI, while imports have the lowest reported contribution. The analysis demonstrates that predictive modelling can be supplemented with variable-importance evidence to provide a more useful interpretation for economic planning. The thesis-based Lagrange extrapolation also projects continued GDP growth through 2030, although such long-range projections should be treated as conditional numerical scenarios.

6. POLICY IMPLICATIONS AND LIMITATIONS

The strong sensitivity to exchange rate suggests that macroeconomic monitoring systems should explicitly incorporate foreign-exchange stability and regime changes when producing GDP forecasts. The high contribution of exports also supports continued attention to export diversification, production capacity, and policies that reduce excessive dependence on a narrow range of export commodities. At the same time, the feature ranking should be interpreted as model-specific predictive importance rather than a causal estimate of economic impact.

The study has limitations. The annual sample is relatively small, and the sensitivity analysis in the present manuscript reports qualitative impact categories rather than the raw permutation-importance distribution. In addition, the thesis documentation contains constructed/simulated-series language in parts of the methodology and limitations sections; this should be made fully consistent with the final data provenance before journal submission. Future work should therefore use longer and higher-frequency series where available, report repeated permutation distributions or confidence intervals, and compare the LIF preprocessing layer with spline, LOESS, or Kalman-filter alternatives. Attention-based models and SHAP-based interpretability could also be investigated.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY

The study uses the annual macroeconomic series documented in the accompanying thesis, with source agencies identified as the Central Bank of Nigeria, National Bureau of Statistics, and World Bank. The

final journal submission should provide the exact data-retrieval links and clarify whether each series is observed, reconstructed, or simulated.

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